

Manufacturing Renaissance Meets Time-Inconsistent Tariffs

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Abstract

We analyze optimal tariffs in a dynamic version of the Venables (1987) framework with a time-to-build lag in firm creation. In this environment, trade protection is driven by the desire to increase firm creation at home at the expense of the foreign trade partner. When firm creation takes time, the benefits of protection accrue in the future, whereas the costs in the form of higher consumer prices occur immediately. Under certain parameterizations, a policymaker that can commit to a full time path of tariffs leaves trade unrestricted at date 0, and announces high tariffs coming on at a pre-specified future period. But there is a time inconsistency: the policymaker's short-run incentive is not to impose contemporaneous tariffs due to the adverse effect on prices. Thus, lack of commitment to future tariffs can preserve free(er) trade and derail a manufacturing renaissance.

Keywords: optimal dynamic tariffs, delocation, time inconsistency

JEL Codes: F02, F12, F13

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1 Introduction

In ushering the new era of protectionism, President Trump’s frequently stated aim is to bring back US manufacturing. The vocabulary includes “new industrialism,” or a coming “manufacturing renaissance.” The language sometimes explicitly calls for delocation of production: “We’re going to take their factories [...]. We’re going to bring thousands and thousands of businesses [...] back to the good old USA.” (Trump, 2024). This would be accomplished by providing market access to the firms that produce domestically: “free access to the best and biggest market on the planet, but only if you make your product here in America” (ibid). However, famously the Trump administration’s first-choice tariffs did not entirely stick. The historic April 2, 2025 announcement was followed by a climbdown only one week later, and many subsequent tariff announcements or threats were similarly followed by various pauses and exemptions.

This paper provides a theory of the failure to stick to high tariffs announced with the objective of a manufacturing renaissance. We argue that it is a manifestation of time inconsistency that occurs naturally in an environment in which delocation takes time. We work with a dynamic version of the Venables (1987) model of tariff policy. There are two large countries. Firms can enter and produce in one of the countries, and export to the other. Trade is subject to both non-tariff and tariff costs. In this environment, each country would like to locate firms in its domestic market, as it saves on trade costs and lowers the domestic price index. One way to induce firms to move to the home country is to raise import tariffs. This model can be viewed as an intellectual underpinning of the type of thinking about tariffs manifested in the quotes above. In the Venables model, the motivation for protection is precisely to delocate firms from abroad to the home country. Tariffs accomplish that by giving firms better market access to the home consumers if they produce domestically.

However, firm delocation takes longer than shipping goods internationally. Thus, we extend the Venables model to a dynamic environment with a time-to-build lag for firm delocation and gradual adjustment of firm location to shocks, as in, e.g., Das, Roberts, and Tybout (2007) and Boehm et al. (2024). To produce in a country, a firm must pay a sunk cost today, but does not begin production until the next period. Separating trade and firm location in time uncovers a tradeoff implicit in the original static Venables (1987) framework: a tariff increases the domestic price index because it makes imports more expensive, but through delocation increases the number of domestic firms, lowering the domestic price index. The first effect occurs instantaneously when the tariff is imposed. The second effect is inoperative instantaneously because of the time-to-build lag, but manifests itself with delay as entry takes place gradually.

We characterize optimal unilateral tariff policy in this environment. In the areas of the parameter space in which the steady state under commitment features high tariffs (which is also the case treated in Venables, 1987), the policymaker wants to use tariffs to induce delocation. But the delocation cannot get underway until the next period, and takes time to reach its full effect. When the policymaker can commit to the full path of tariffs, the optimal policy features free trade (or low tariffs) for the initial few periods, followed by a gradual convergence to a permanent high tariff

preannounced to come on at a specified future date. This optimal policy accomplishes the delocation objective, as firms' decisions are forward-looking. Knowing that the tariffs will be high in the future, firms today pay the sunk costs to locate in the home country. At the same time, sticking to free trade in the present period accomplishes the goal of keeping the consumer prices low today.¹

The temporal misalignment of the costs and benefits of tariffs also implies that the optimal time path of tariffs under commitment is generally time-inconsistent. In any period, the cost of higher consumer prices coupled with fixed firm locations leads the policymaker to prefer contemporaneous free trade. Thus, regardless of the pre-announced time path of tariffs, when the policymaker arrives to the periods in which tariffs should increase, she instead finds it optimal to continue free trade.

It is in this sense that in a dynamic Venables economy, time inconsistency coupled with lack of commitment derails the manufacturing renaissance. The immediately incurred pain of higher domestic prices without a contemporaneous benefit of a domestic manufacturing boom creates a temptation to back off from high tariffs. Indeed, higher domestic prices were the apparent motivation for some prominent tariff exemptions, such as the exemption on the import of smartphones, laptops, and other consumer electronics from China that went into effect only 10 days after the April 2, 2025 announcement (Mickle and Swanson, 2025).²

While the model produces the inability to stick to optimally high long-run tariffs under some parameterizations, under different parameter values it also features a variety of alternative dynamic optimal tariff profiles. Two crucial parameters are (i) the fraction of tariff revenue returned to households and used for consumption and (ii) the level of the non-tariff iceberg trade cost. In the original treatments of the delocation motive by Venables (1987) and Ossa (2011), tariffs do not generate useable revenue (in fact, they are modeled as iceberg trade costs). In our framework, the fraction of tariff revenue that is rebated to households lump-sum is governed by a parameter. When that fraction is 0, the model is close to the pure Venables delocation benchmark. When that fraction is 1, the model matches the commonly adopted assumption in the optimal tariff literature that none of the tariff revenue is wasted.

The clearest parameterization in which trade is free in the short run while the delocation force dominates in the long run is when the fraction of tariff revenue rebated to households is low, and the iceberg trade costs are moderate to high. The delocation motive is most powerful when trade costs are substantial, since in that case every firm created at home at the expense of shutting down a firm abroad can serve the home market with a lower price. When tariff revenues are not rebated to households, the motive to impose tariffs in the present period is muted.

However, under different parameter values the optimal tariff profile under commitment can reverse. An example is a parameterization where iceberg costs are negligible and tariff revenue is

¹To be precise, the free trade outcome in the present obtains under the assumption, also adopted in Venables (1987), that a sufficiently small fraction of tariff revenue is rebated to households and used for consumption. As will become clear below, free trade in the first period is not a necessary outcome under all parameter values.

²Note that in this environment the temptation to go to free trade occurs regardless of the tariffs that the policymaker inherits from the previous period. Even if the initial equilibrium features prohibitive tariffs, the temptation to lower tariffs for the present period is still there.

fully rebated to households for consumption. Without iceberg trade costs, the delocation motive is weakest, as it is less important where in the world the firms are located. When the revenue motive is powerful, the optimal tariff under commitment instead becomes akin to the [Chamley \(1986\)](#)-[Judd \(1985\)](#) dynamic optimal capital taxation problem – since foreign firms are a form of capital. In this case, the commitment tariff path starts at a higher initial value and converges to a lower steady state value. In the initial period, the stock of foreign firms is fixed, and it is optimal to tax (tariff) them to generate revenue. However, taxing foreign firms is costly in the long run, as it discourages foreign entry (i.e. capital accumulation) and erodes the tax base. Thus, the steady-state tariff is low. In this case, dynamic inconsistency manifests itself as an inability to stick to the promise of low tariffs. When the delocation motive is muted, the optimal import tariff problem has the well-known features of the dynamically inconsistent capital taxation problem ([Kydland and Prescott, 1977, 1980](#); [Fischer, 1980](#)), also noted recently in the context of optimal tariffs by [Bassetto, Huo, and Ríos-Rull \(2024\)](#) and [Alessandria et al. \(2026\)](#).

Related literature Our paper draws from, and contributes to, two literatures. The first is the theory of optimal trade protection in the presence of delocation motives pioneered by [Venables \(1987\)](#) (see, among others, [Helpman and Krugman, 1989](#); [Demidova and Rodríguez-Clare, 2009](#); [Ossa, 2011, 2014](#); [Felbermayr, Jung, and Larch, 2013](#); [Campolmi, Fadinger, and Forlati, 2014](#); [Bagwell and Staiger, 2015](#); [Haaland and Venables, 2016](#)). We develop a dynamic version of this framework, and show that doing so carries rich implications. We thus contribute to the recently emerging body of work on dynamic optimal tariffs ([Bassetto, Huo, and Ríos-Rull, 2024](#); [Bai et al., 2025](#); [Dávila et al., 2025](#); [Alessandria, Ding, and Khan, 2025](#); [Alessandria et al., 2026](#); [Auray, Devereux, and Eyquem, 2026](#)). These papers point out that in dynamic environments tariff policy features time inconsistency, a property long noted in the trade policy literature (e.g. [Eaton and Grossman, 1985](#); [Staiger and Tabellini, 1987](#); [Matsuyama, 1990](#)). Our paper studies this inconsistency in a setting in which tariffs are driven by the delocation motive rather than the terms of trade. We characterize the parameter configurations under which lack of commitment manifests itself in the inability to stick to high tariffs. Lack of commitment has also been studied as a motive for trade agreements by [Maggi and Rodríguez-Clare \(1998; 2007\)](#). In those papers, the government uses trade agreements as a commitment device against the influence of special interest groups. In our framework the policymaker is benevolent, and lack of commitment manifests itself dynamically as time inconsistency. More broadly, our paper belongs to the optimal tariff literature that has been active since the 2016 trade regime change (see, among others, [Lashkaripour, 2021](#); [Caliendo et al., 2023](#); [Caliendo and Feenstra, 2024](#); [Bai et al., 2025](#); [Bai, Lu, and Wang, 2025](#); [Becko, Grossman, and Helpman, 2025](#); [Hu, Mei, and Ni, 2025](#); [Ignatenko et al., 2025](#); [Itskhoki and Mukhin, 2025](#); [Blanchard, Bown, and Johnson, 2026](#)).

Roadmap Section 2 describes the model economy and its properties. Section 3 sets up the dynamic optimal tariff problem in this environment and characterizes the optimal tariffs. It also de-

scribes the Markov equilibrium that obtains without commitment. Section 4 concludes. Detailed derivations and most proofs are collected in the Appendix.

2 Model

2.1 The environment

We consider a dynamic variant of the [Venables \(1987\)](#) economy, which is in turn based on the [Krugman \(1980\)](#) monopolistic competition model of international trade. The world is comprised of 2 countries, 1 and 2, indexed by i and j . There is one factor of production, labor, with country endowments L_j . There are two sectors, a manufacturing sector m and a homogeneous good sector h . The manufacturing sector is populated by monopolistically competitive firms. The homogeneous good is produced competitively, freely traded and serves as the numeraire. Time is discrete and indexed by t . There is no aggregate risk, and no equilibrium trade in assets.

Households The representative household in country $j \in \{1, 2\}$ has quasi-linear flow utility and discounts the future with discount factor $\beta \in (0, 1)$. The household maximizes

$$\max_{\{C_{jt}^m, C_{jt}^h, B_{jt}\}} \sum_{t=0}^{\infty} \beta^t (\ln C_{jt}^m + C_{jt}^h)$$

subject to the budget constraint

$$P_{jt}C_{jt}^m + C_{jt}^h + B_{jt} = w_{jt}L_j + \Pi_{jt} + \rho R_{jt} + B_{jt-1} (1 + r_{jt-1}) \quad (1)$$

and a no-Ponzi-game condition. Here C_{jt}^m denotes consumption of the manufacturing good, C_{jt}^h is consumption of the homogeneous good, w_{jt} is the nominal wage, and Π_{jt} aggregate profits. The bond B_{jt} is not traded in equilibrium so that $B_{jt} = 0$; we include it in the budget constraint only to price the interest rate r_{jt} . The term R_{jt} denotes tariff revenues, of which the share $\rho \in [0, 1]$ is rebated to households. The remainder is spent by the government on homogeneous goods in a way that the consumer does not value. As will become clear below, parameter ρ is important. It governs the strength of the revenue motive behind imposing tariffs. Treating it as a parameter allows us to nest the various approaches adopted in the literature. When $\rho = 1$, the entirety of tariff revenues is rebated lump sum to the households and is used for consumption. This corresponds to the conventional assumption in many optimal tariff exercises, such as [Gros \(1987\)](#) and the literature that follows in this tradition. When $\rho = 0$, the tariff revenue is “thrown in the ocean,” and thus the revenue motive for imposing tariffs is inoperative. This assumption is adopted in the firm-delocation optimal tariff literature of [Venables \(1987\)](#) and [Ossa \(2011\)](#).

We assume throughout that countries’ labor endowments L_j are sufficiently large that both countries produce and consume the homogeneous good in equilibrium. Optimal behavior of the

household (and imposing that in equilibrium $B_{jt} = 0$) implies:

$$\frac{1}{C_{jt}^m} = P_{jt}, \quad (2)$$

$$C_{jt}^h = w_{jt}L_j + \Pi_{jt} + \rho R_{jt} - 1, \quad (3)$$

$$\frac{1}{1 + r_{jt}} = \beta. \quad (4)$$

Aggregating firm A perfectly competitive aggregating firm combines varieties from potentially all countries i serving market j to produce the manufacturing basket C_{jt}^m using the CES aggregation technology with elasticity $\sigma > 1$:

$$C_{jt}^m = \left(\sum_i \int_{\Omega_{it-1}} q_{ijt}(\omega) \left(\frac{p_{ijt}^c(\omega)}{P_{jt}} \right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}.$$

In this expression $q_{ijt}(\omega)$ is the quantity supplied by firm ω from country i to market j and the set Ω_{it-1} represents all varieties sold by firms from country i at time t . Since this set is determined in the previous period, it is indexed $t - 1$. (Throughout, for variables varying bilaterally, the first subscript denotes the source, the second the destination.) Optimal behavior implies the demand curve

$$q_{ijt}(\omega) = C_{jt}^m \left(\frac{p_{ijt}^c(\omega)}{P_{jt}} \right)^{-\sigma}, \quad (5)$$

where $p_{ijt}^c(\omega)$ is the price of variety ω from i faced by consumers in country j , and P_{jt} the manufacturing price index in country j :

$$P_{jt} = \left(\sum_i \int_{\Omega_{it-1}} \left(p_{ijt}^c(\omega) \right)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}. \quad (6)$$

Manufacturing firms Manufacturing firms in country i use labor to produce differentiated goods they sell in their own market and the foreign market. The technology for serving their own and the foreign market, respectively, is

$$q_{ijt}(\omega) = \frac{l_{ijt}(\omega)}{\theta_{ij}c} \quad \text{where} \quad \theta_{ij} = \begin{cases} 1 & i = j \\ \theta \geq 1 & i \neq j \end{cases}, \quad (7)$$

$l_{ijt}(\omega)$ is the labor employed by ω , c is the unit labor requirement, and θ is the symmetric iceberg trade cost that applies to all cross-border shipments. The parameter θ is important, because it affects the strength of the delocation motive. Higher trade costs increase the relative benefit of an additional firm in the home market, compared to an additional firm in the foreign market.

Governments impose gross ad-valorem tariffs $\tau_{ijt} \geq 1$ on shipments from abroad ($i \neq j$). These

tariffs drive a wedge between the price $p_{ijt}^c(\omega)$ that country j consumers pay and the price $p_{ijt}^x(\omega)$ that sellers in country i receive: $p_{ijt}^c(\omega) = \tau_{ijt} p_{ijt}^x(\omega)$. We assume there are no domestic taxes, $\tau_{jjt} = 1$.

Manufacturing firms are monopolistically competitive. In every period t and conditional on operating, manufacturing firms in country i solve

$$\max_{\{p_{ijt}^x(\omega), q_{ijt}(\omega), l_{ijt}(\omega)\}_j} \sum_j \left[p_{ijt}^x(\omega) q_{ijt}(\omega) - w_{it} l_{ijt}(\omega) \right]$$

subject to the technology (7) and the demand curve (5) with $p_{ijt}^c(\omega) = \tau_{ijt} p_{ijt}^x(\omega)$. At their optimum, firms set prices at constant markups over marginal costs:

$$p_{ijt}^x(\omega) = p \theta_{ij} w_{it}, \quad (8)$$

where $p = \frac{\sigma}{\sigma-1} c$. Note that these prices are common across firms within a country pair. Therefore, whenever a variable is common across firms, we omit the firm index ω to streamline notation. Operating profits per firm in country i are

$$\pi_{it} = \frac{w_{it} c}{\sigma - 1} \left[C_{it}^m \left(\frac{p w_{it}}{P_{it}} \right)^{-\sigma} + \theta C_{jt}^m \left(\frac{p \tau_{ijt} \theta w_{it}}{P_{jt}} \right)^{-\sigma} \right]. \quad (9)$$

The entry problem follows [Boehm et al. \(2024\)](#). In every period there is a unit mass of potential entrants. Each entrant ω draws an independently and identically distributed sunk cost $\xi_{it}(\omega)$ denominated in units of domestic labor. Crucially, there is a one-period time-to-build: if a firm pays the sunk costs of entry in period t , it only begins receiving operating profits from the next period onward until it exits. Exit is random and occurs with probability δ . The value of entering is therefore

$$v_{it} = \frac{1}{1 + r_{it}} (\pi_{it+1} + (1 - \delta) v_{it+1}). \quad (10)$$

A firm enters if the value of entry is greater than its sunk entry costs, $v_{it} \geq w_{it} \xi_{it}(\omega)$, so the marginal firm's sunk cost $\bar{\xi}_{it}$ satisfies

$$v_{it} = w_{it} \bar{\xi}_{it}. \quad (11)$$

Let G denote the cumulative distribution function of sunk costs. Then the mass of firms in country i evolves according to the law of motion

$$n_{it} = (1 - \delta) n_{it-1} + G(\bar{\xi}_{it}). \quad (12)$$

Since all firms behave identically post-entry, the mass of firms n_{it} is sufficient to characterize the set of firms in country i , Ω_{it} .

We assume that manufacturing firms draw entry costs from the inverse Pareto distribution:

$$G(\xi) = (b\xi)^\chi$$

for $0 \leq \xi \leq \frac{1}{b}$ and $b, \chi > 0$. The parameter χ is important for what follows. It corresponds to the elasticity of the entry response to the value of the firm:

$$\frac{\partial \ln G(v_t)}{\partial \ln v_t} = \frac{\chi b^\chi v_t^{\chi-1} v_t}{(bv_t)^\chi} = \chi.$$

As $\chi \rightarrow 0$, the mass of firms becomes fixed. As $\chi \rightarrow \infty$, we converge to the classic free entry setup of [Krugman \(1980\)](#). This setup allows us to flexibly parameterize the strength of the entry response to tariffs.

Homogeneous goods sector Firms in the homogeneous goods sector take prices as given, operate technology $Q_{it}^h = l_{it}^h$, and maximize profits. In any equilibrium with strictly positive and finite employment in the homogeneous goods sector, it must therefore be that

$$w_{it} = 1. \tag{13}$$

Profits and tariff revenues Aggregate profits in country j at time t are

$$\Pi_{jt} = n_{jt-1} \pi_{jt} - w_{jt} b^\chi \frac{\chi}{\chi+1} (\bar{\xi}_{jt})^{\chi+1}, \tag{14}$$

where the last term captures the sunk costs $w_{jt} \int_0^{\bar{\xi}_{jt}} \xi dG(\xi)$ paid by firms entering at t . Tariff revenues in country j are

$$R_{jt} = (\tau_{ijt} - 1) n_{it-1} p_{ijt}^x q_{ijt} \quad \text{for } i \neq j. \tag{15}$$

Market clearing In addition to bond market clearing $B_{jt} = 0, j \in \{1, 2\}$, the market for homogeneous goods clears,

$$\sum_j (C_{jt}^h + (1 - \rho) R_{jt}) = \sum_j Q_{jt}^h,$$

and the labor markets of both countries clear, so that

$$L_j = n_{jt-1} \sum_i l_{jit} + l_{jt}^h + b^\chi \frac{\chi}{\chi+1} (\bar{\xi}_{jt})^{\chi+1}$$

for $j \in \{1, 2\}$.

Expenditure shares Denote by s_{ijt} the share of goods from i in country j 's manufacturing expenditure. For what follows, it will be convenient to state the expressions for these expenditure

shares:

$$\begin{aligned} s_{11t} &= n_{1t-1} p^{1-\sigma} P_{1t}^{\sigma-1}, & s_{21t} &= n_{2t-1} (\theta \tau_{21t} p)^{1-\sigma} P_{1t}^{\sigma-1}, \\ s_{22t} &= n_{2t-1} p^{1-\sigma} P_{2t}^{\sigma-1}, & s_{12t} &= n_{1t-1} (\theta \tau_{12t} p)^{1-\sigma} P_{2t}^{\sigma-1}. \end{aligned} \quad (16)$$

Note that $s_{11t} + s_{21t} = s_{12t} + s_{22t} = 1$ and that

$$R_{1t} = (1 - \tau_{21t}^{-1}) s_{21t}.$$

These further imply the following relationships between operating profits, tariffs, and shares:

$$\sigma n_{1t-1} \pi_{1t} = s_{11t} + \frac{s_{12t}}{\tau_{12t}}, \quad \sigma n_{2t-1} \pi_{2t} = s_{22t} + \frac{s_{21t}}{\tau_{21t}}.$$

Equilibrium

Definition 1 Given a path of tariffs $\{\tau_{12t}, \tau_{21t}\}_{t \geq 0}$ and initial conditions for the masses of firms, $n_{1,-1}$ and $n_{2,-1}$, an **equilibrium** in this economy is a set of prices $\{P_{jt}, w_{jt}, r_{jt}\}_{t \geq 0}$, quantities $\{C_{jt}^h, C_{jt}^m, q_{ijt}, l_{jt}^h, l_{ijt}\}_{t \geq 0}$, operating profits, masses, and values of firms $\{\pi_{jt}, n_{jt}, v_{jt}\}_{t \geq 0}$ such that

- i. Taking prices as given, consumers in both countries solve their optimization problem (equations 2-4);
- ii. Taking prices as given, the aggregating firms maximize profits (equations 5-6);
- iii. Taking demand curves as given, manufacturing firms maximize profits and enter optimally (equations 8-11);
- iv. Taking prices as given, homogeneous goods firms maximize profits (equation 13);
- v. The masses of firms evolve according to equation (12) for $i \in \{1, 2\}$.
- vi. Domestic bond markets clear, domestic labor markets clear, the market for homogeneous goods clears, and the markets for manufacturing goods all clear.

We restrict attention to parameters and tariff paths for which firm entry is interior. Combining the inverse Pareto cdf with equations (11) and (13) shows that doing so requires that $0 < v_{it} < 1/b$.

2.2 Some properties of the economy

Before setting up and solving the policymaker's tariff-setting problem, we describe some properties of this economy, that will clarify the results that follow.

Lemma 1 (*Global manufacturing expenditures, sales, and operating profits*) For all t

- i. global consumer manufacturing expenditures are

$$n_{1t-1} (p_{11t}^c q_{11t} + p_{12t}^c q_{12t}) + n_{2t-1} (p_{22t}^c q_{22t} + p_{21t}^c q_{21t}) = 2.$$

ii. global manufacturing producer revenues are

$$n_{1t-1} (p_{11t}^x q_{11t} + p_{12t}^x q_{12t}) + n_{2t-1} (p_{22t}^x q_{22t} + p_{21t}^x q_{21t}) = 2 - R_{1t} - R_{2t}.$$

iii. global operating profits are

$$n_{1t-1} \pi_{1t} + n_{2t-1} \pi_{2t} = \frac{1}{\sigma} [2 - R_{1t} - R_{2t}].$$

Proof. See Appendix B. ■

Item (i) of the lemma says that the worldwide size of manufacturing expenditures is constant. It is a direct consequence of the quasi-linear utility assumption. Items (ii) and (iii) say that worldwide manufacturing sales and operating profits are proportional to the constant consumer expenditure minus the global tariff revenue. This property is useful because it highlights — for fixed revenues $R_{1t} + R_{2t}$ — the negative relationship between countries operating profits, and therefore firm masses, between the two countries. This negative relationship is at the heart of the delocation motive for tariffs in Venables (1987). The following lemma establishes this more precisely.

Lemma 2 (Manufacturing firms and delocation) *In steady state*

$$n_1^{\frac{\chi+1}{\chi}} + n_2^{\frac{\chi+1}{\chi}} = \frac{1}{\delta^{\frac{1}{\chi}}} \frac{b\beta}{1 - \beta(1 - \delta)} \frac{1}{\sigma} [2 - R_1 - R_2].$$

Proof. See Appendix B. ■

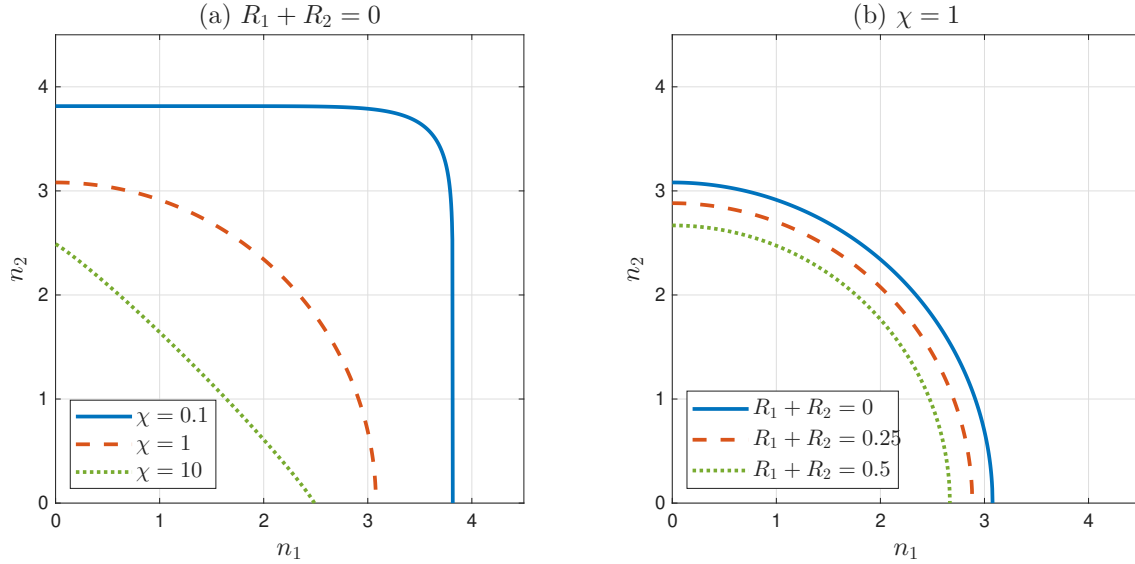
Lemma 2 states that the sum of two convex functions of n_1 and n_2 , respectively, equals a constant times producer operating profits $\frac{1}{\sigma} [2 - R_1 - R_2]$. It generalizes the Venables (1987)-Ossa (2011) property, which obtains under free entry. As $\chi \rightarrow \infty$, the economy features one-for-one crowding out:

$$n_1 + n_2 = \frac{b\beta}{1 - \beta(1 - \delta)} \frac{1}{\sigma} [2 - R_1 - R_2],$$

in which raising n_1 by one unit reduces n_2 by one unit (holding the RHS constant). More generally, χ parameterizes the magnitude of the delocation effect. The left panel of Figure 1 plots n_1 and n_2 under different values of χ . The relationship is always negative, but becomes more linear as χ increases. By contrast, for small χ the numbers of firms are quasi-fixed.

In addition, the lemma shows that higher overall tariff revenues reduce firm operating profits and drive down this index of the masses of firms. Because tariffs were modeled as iceberg costs by Venables (1987) and parts of Ossa (2011), this property does not appear in those papers. In our model tariff revenues reduce the worldwide numbers of firms, as illustrated in the right panel of Figure 1. Of course, the lemma is not a characterization of how the masses of firms in each country behave as a function of primitives or tariffs in the 2 countries. And an equilibrium for a given set of tariffs and primitives will be a single point on these curves. Nonetheless these properties illustrate

Figure 1: Masses of firms in the two countries



Notes: The figure plots n_2 against n_1 for different χ (left panel) and different levels of overall revenue $R_1 + R_2$ (right panel). The remaining parameter values are $\sigma = 3$, $\delta = 0.25$, $b = 1$, $\beta = 0.97$.

both the logic of delocation – masses of firms in the two countries are negatively related – and the impact of tariff revenues on profits and firms.

Lemma 3 (Global profits) *In steady state, global profits in the economy are*

$$\Pi_1 + \Pi_2 = \left(1 - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)}\right) \frac{1}{\sigma} [2 - R_1 - R_2].$$

Proof. See Appendix B. ■

Global profits inclusive of sunk costs also scale with $\frac{1}{\sigma} [2 - R_1 - R_2]$, the global manufacturing variable profits (see Lemma 1). The constant $\left(1 - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)}\right)$ captures how much of operating profits is left after sunk cost payments. Like Lemma 2, the implication here is that tariffs have two effects. They change the size of the pie and they change how profits are distributed across countries.

Corollary 1 (Global household income) *In steady state, global household income in the economy is*

$$L_1 + L_2 + \frac{2}{\sigma} \left(1 - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)}\right) - \frac{1}{\sigma} \left(1 - \sigma\rho - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)}\right) (R_1 + R_2).$$

Global household income is nondecreasing in global tariff revenues if and only if $\rho \geq \rho^*$, where

$$\rho^* = \frac{1}{\sigma} \left(1 - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)}\right).$$

Proof. See Appendix B. ■

Tariffs change the size of the pie (now global household income) and lead to redistribution. Unlike manufacturing firm profits, global household income can be decreasing or increasing in global tariff revenues $R_1 + R_2$, depending on ρ . This is because tariff revenues cut into profits (see Lemma 3). If $\rho < \rho^*$, then the reduction in profits is greater than the increase in transfers that households receive. If $\rho > \rho^*$, then the reduction in profits is smaller than the increase in household transfers. For $\rho = \rho^*$, global household income is independent of tariff revenues.

3 Dynamic optimal tariffs

This section characterizes the optimal unilateral tariff of this economy. We assume that the benevolent policymaker in country 1 solves the optimal policy problem of setting a path of tariffs $\{\tau_{21t}\}_{t=0}^{\infty}$ to maximize the present value of household welfare. Throughout, the foreign tariff path $\{\tau_{12t}\}_{t=0}^{\infty}$ is taken as given, and allowed to vary over time. After setting up the generic policy problem, we begin with the optimal policy under discretion. We analyze the Markov perfect equilibrium in a game in which the policymaker cannot commit to a future tariff path and takes her own future policy choices as given. We show that in this case, the optimal policy problem becomes simple and static as the policymaker perceives the evolution of firms to be exogenous to policy. As a result the two main motives for setting tariffs are revenue generation (for $\rho > 0$) and the adverse effects on consumer prices.

We then move on to optimal policy under commitment. If the policymaker has the ability to commit, she will announce a tariff path at $t = 0$ and implement it as announced at every future date. Under commitment, a third motive for setting tariffs becomes relevant, as the policymaker understands that by committing to a particular tariff path she can influence the masses of foreign and domestic firms through their entry decisions. We partially characterize the welfare-maximizing path of tariffs analytically. This analysis highlights the intertemporal tradeoffs inherent in the environment in which firm entry is a forward-looking decision subject to time-to-build. Generically, the optimal tariff path for $t > 0$ will differ from the optimal commitment tariff at $t = 0$ (and under discretion), leading to a temptation for the policymaker to re-optimize policy at a future date and thus establishing the presence of a time inconsistency problem.

Third, we discuss the nature of this time-inconsistency. When ρ is low and θ takes intermediate values, the long-run steady-state optimal tariff under commitment is dominated by the Venables firm-delocation motive: with costly trade the country benefits from hosting as many firms as possible. Since the numbers of domestic and foreign firms have a negative relationship (Lemma 2), steady-state tariffs are high because of the desire to suppress foreign entry and increase domestic entry. However, the delocation motive is absent at $t = 0$ because within a period, the masses of firms are predetermined and fixed (assuming no initial promises). Without the contemporaneous delocation motive, the $t = 0$ tariff is low, and thus the optimal path under commitment is to keep tariffs low at $t = 0$ but announce a high future tariff. Without commitment, though the high

tariff is desirable in the long run, the temptation to set low tariffs every period implies that the policymaker is unable to stick to the promise of high tariffs.

By contrast, when ρ is high and θ low, the commitment path has some features of the Chamley-Judd optimal capital taxation problem. Foreign firms are a form of accumulable capital. Thus, the steady state tariff is low in order to avoid choking off foreign firm investment and prevent the erosion of the tax base. However, at $t = 0$ the tax base – determined in part by the set of foreign firms – is predetermined, so the optimal policy is to tax it at a higher rate than in steady state. The optimal commitment path features a higher tariff at $t = 0$, coupled with a promise of low tariffs in steady state. Without commitment, however, the policymaker cannot stick to the promise of low future tariffs, and continues to impose high statically optimal tariffs.

The bottom line is that the time inconsistency in the dynamic optimal tariff problem is pervasive. Whether it manifests itself in the inability to impose high tariffs, or the inability to impose low tariffs, depends on whether the firm delocation motive or the revenue motive is dominant.

3.1 The generic policy problem

Quasi-linear preferences imply unit expenditure on manufactures. Household welfare therefore depends on the manufacturing price index, income from profits of domestic firms net of entry costs, and rebated tariff revenue. The following lemma expresses this welfare and the equilibrium conditions of Section 2 in terms of the variables we use to study optimal policy. Throughout, we assume that, given the inherited firm stocks, the private equilibrium is unique for any tariff path.

Lemma 4 (The Ramsey problem) *Maintain the assumptions of Section 2, including positive production and consumption of the homogeneous good and interior entry ($0 < bv_{it} < 1$), for both countries at every t . Assume $\sigma > 1$, $\theta \geq 1$, $\beta, \delta \in (0, 1)$, $\chi > 0$, and $b, p > 0$. Given positive initial firm stocks $n_{1,-1}, n_{2,-1} > 0$ and the foreign tariff path $\{\tau_{12t}\}_{t=0,\dots,\infty}$, the policymaker in country 1 chooses the domestic tariff path $\{\tau_{21t}\}_{t=0,\dots,\infty}$ to maximize:*

$$\tilde{U}_1 = \sum_{t=0}^{\infty} \beta^t \left[-\ln P_{1t} + n_{1t-1} \pi_{1t} - b^\chi \frac{\chi}{\chi+1} v_{1t}^{\chi+1} + \rho \left(1 - \frac{1}{\tau_{21t}} \right) \left(1 - P_{1t}^{\sigma-1} n_{1t-1} p^{1-\sigma} \right) \right], \quad (17)$$

subject to

$$\begin{aligned}
P_{1t}^{1-\sigma} &= n_{1t-1}p^{1-\sigma} + n_{2t-1}(\theta\tau_{21t}p)^{1-\sigma} & \lambda_{1t}^P, \\
P_{2t}^{1-\sigma} &= n_{2t-1}p^{1-\sigma} + n_{1t-1}(\theta\tau_{12t}p)^{1-\sigma} & \lambda_{2t}^P, \\
\pi_{1t} &= \frac{p^{1-\sigma}}{\sigma} \left(P_{1t}^{\sigma-1} + P_{2t}^{\sigma-1}\tau_{12t}^{-\sigma}\theta^{1-\sigma} \right) & \lambda_{1t}^\pi, \\
\pi_{2t} &= \frac{p^{1-\sigma}}{\sigma} \left(P_{2t}^{\sigma-1} + P_{1t}^{\sigma-1}\tau_{21t}^{-\sigma}\theta^{1-\sigma} \right) & \lambda_{2t}^\pi, \\
n_{it} &= (1-\delta)n_{it-1} + (bv_{it})^\chi, \quad i = 1, 2 & \lambda_{it}^n, \\
v_{it} &= \beta [\pi_{it+1} + (1-\delta)v_{it+1}], \quad i = 1, 2 & \lambda_{it}^v, \\
\tau_{21t} &\in [1, \infty) & \lambda_t^{\tau_{LB}}.
\end{aligned}$$

Proof. See Appendix A.1. ■

Household life-time utility can be expressed as:

$$\sum_{t=0}^{\infty} \beta^t \left(\ln C_{1t}^m + C_{1t}^h \right) = \tilde{U}_1 + \frac{L_1 - 1}{1 - \beta}. \quad (18)$$

The constant in (18) has no effect on the ranking of tariff paths. Prices, profits, firm stocks, and firm values in this formulation are equilibrium outcomes; the government chooses the domestic tariff path. The second and third terms in the maximand are domestic firms' profits: $\Pi_{1t} = n_{1t-1}\pi_{1t} - b\chi \frac{\chi}{\chi+1} v_{1t}^{\chi+1}$. The last term is the part of the tariff revenue rebated to households: $\rho R_{1t} = \rho \left(1 - \frac{1}{\tau_{21t}} \right) \left(1 - P_{1t}^{\sigma-1} n_{1t-1} p^{1-\sigma} \right)$. The notation for the Lagrange multipliers is stated next to each constraint. Note that we constrain the tariffs to be bounded from below by 1 (no import subsidies).

3.2 Optimal policy under discretion

We begin our analysis of policy with the case of discretion. This case highlights the “static” incentives that the policymaker faces. The revenue motive provides an incentive to set higher tariffs. By contrast, the adverse effect on the price index implies lower optimal tariffs. A third motive — that higher tariffs raise profits of domestic firms — is present, but turns out to always be dominated by the effect on the price index. To analyze the optimal tariff under discretion, we therefore study the Markov-perfect equilibrium.

Definition 2 (The Markov perfect equilibrium) Given $\{\tau_{12t}\}_{t=0}^{\infty}$, a Markov-perfect equilibrium consists of a policy rule ψ and an allocation such that (i) the allocation satisfies the equilibrium conditions above at every date with $\tau_{21t} = \psi(n_{1t-1}, n_{2t-1})$, and (ii) for every date t and every (n_{1t-1}, n_{2t-1}) , $\psi(n_{1t-1}, n_{2t-1})$ maximizes objective (17) from date t onward over $\tau_{21t} \in [1, \infty)$, taking as given (n_{1t-1}, n_{2t-1}) and that ψ is followed at every date after t .

As before, we assume that, given (n_{1t-1}, n_{2t-1}) , any date- t tariff, and the rule ψ at later dates,

the private continuation equilibrium is unique. We next show that the policy problem drastically simplifies in the case of the Markov-perfect equilibrium.

Lemma 5 (Time t tariffs and future allocations) *Fix a date t , a state (n_{1t-1}, n_{2t-1}) , and a rule ψ that is followed from date $t + 1$ onward. Then n_{1t} , n_{2t} , v_{1t} , v_{2t} , and the entire allocation from date $t + 1$ onward are independent of τ_{21t} .*

Proof. Suppose first that (n_{1t}, n_{2t}) are fixed and known. From date $t + 1$ onward the domestic tariff is $\tau_{21k} = \psi(n_{1k-1}, n_{2k-1})$, $k \geq t + 1$, so, by the uniqueness assumption stated earlier, the equilibrium conditions uniquely determine $\{P_{ik}, \pi_{ik}, n_{ik}, v_{ik}\}_{k \geq t+1}$ as functions of (n_{1t}, n_{2t}) and $\{\tau_{12k}\}_{k \geq t+1}$ alone; the date- t tariff appears in none of these equations. Next return to the initial pair (n_{1t}, n_{2t}) . The values v_{1t} and v_{2t} , which satisfy $v_{it} = \beta(\pi_{it+1} + (1 - \delta)v_{it+1})$ and hence involve only date $t + 1$ variables, are only functions of (n_{1t}, n_{2t}) , ψ , and $\{\tau_{12k}\}_{k \geq t+1}$. Combining this with

$$n_{1t} = (1 - \delta)n_{1t-1} + (bv_{1t})^\chi, \quad n_{2t} = (1 - \delta)n_{2t-1} + (bv_{2t})^\chi$$

gives four equations in $(n_{1t}, n_{2t}, v_{1t}, v_{2t})$ whose only other arguments are (n_{1t-1}, n_{2t-1}) , ψ , and $\{\tau_{12k}\}_{k \geq t+1}$. Since τ_{21t} does not enter and the solution is unique, it is independent of τ_{21t} . ■

The content of the lemma is a timing property. The date- t tariff changes date- t profits, but date- t profits accrue to the masses n_{1t-1} and n_{2t-1} of firms that entered in the past. Entry at date t is governed by v_{1t} and v_{2t} , the value of a firm that begins to earn profits at $t + 1$, which the date- t tariff cannot affect under discretion, because the future tariff choice is taken as given.

Proposition 1 (The Markov perfect equilibrium) *In any Markov-perfect equilibrium, for every date t and every (n_{1t-1}, n_{2t-1}) , the tariff $\tau_{21t} = \psi(n_{1t-1}, n_{2t-1})$ solves the static problem*

$$\max_{\tau_{21t} \in [1, \infty)} -\ln P_{1t} + n_{1t-1}\pi_{1t} + \rho \left(1 - \tau_{21t}^{-1}\right) \left(1 - P_{1t}^{\sigma-1}n_{1t-1}p^{1-\sigma}\right)$$

subject to

$$P_{1t}^{1-\sigma} = n_{2t-1}(\tau_{21t}\theta p)^{1-\sigma} + n_{1t-1}(p)^{1-\sigma}$$

$$P_{2t}^{1-\sigma} = n_{1t-1}(\tau_{12t}\theta p)^{1-\sigma} + n_{2t-1}(p)^{1-\sigma}$$

$$\pi_{1t} = \frac{1}{\sigma}p^{1-\sigma} \left(P_{1t}^{\sigma-1} + P_{2t}^{\sigma-1}\tau_{12t}^{-\sigma}\theta^{1-\sigma}\right).$$

Proof. By part (ii) of Definition 2, τ_{21t} maximizes the date- t continuation payoff taking (n_{1t-1}, n_{2t-1}) as given. By Lemma 5, every term dated $k \geq t + 1$ is independent of τ_{21t} , and so is v_{1t} and therefore the date- t entry cost term $b^\chi \frac{\chi}{\chi+1} (v_{1t})^{\chi+1}$ is independent of policy. What remains are the date- t terms displayed above, which depend on τ_{21t} only through P_{1t} and π_{1t} . ■

Three remarks are in order. First, the only dynamic consequence of the date- t tariff runs through v_{1t-k} and v_{2t-k} , $k \geq 1$, that is, through entry at date $t - 1$ and before. A policymaker with commitment internalizes this backward channel; the date- t policymaker under discretion

does not. This turns out to be the sole source of time inconsistency here, and it is also why the discretionary problem collapses to a sequence of static problems while the commitment problem does not.

Second, the reduction does not require the environment to be stationary. A time-varying foreign tariff makes the mapping from (n_{1t-1}, n_{2t-1}) into the continuation allocation date-dependent, but Lemma 5 and Proposition 1 apply date by date. Moreover P_{2t} does not depend on τ_{21t} and enters π_{1t} additively, so τ_{12t} drops out of the first-order condition derived below. The discretionary rule ψ is therefore the same function of (n_{1t-1}, n_{2t-1}) at every date, even when the foreign tariff varies over time.

Third, while tariffs levied by country 1 affect country 2 profits (π_{2t}), these profits enter none of the equations relevant for the discretionary policymaker. The constraint can therefore be dropped.

We now discuss the optimal policy under discretion. The marginal effect of tariffs on period- t welfare contains the effects on manufacturing prices, domestic operating profits, and rebated revenue:

$$\begin{aligned} \frac{\partial}{\partial \tau_{21t}} & \left[-\ln P_{1t} + n_{1t-1}\pi_{1t} + \rho(1 - \tau_{21t}^{-1})s_{21t} \right] \\ &= \underbrace{-\frac{s_{21t}}{\tau_{21t}}}_{\text{Consumer prices}} + \underbrace{\frac{(\sigma - 1)s_{11t}s_{21t}}{\sigma\tau_{21t}}}_{\text{Home profits}} + \underbrace{\frac{\rho s_{21t}}{\tau_{21t}^2} [1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}]}_{\text{Rebated revenue}}. \end{aligned} \quad (19)$$

The consumer price index contribution is negative and the home profit contribution is positive. Since $(\sigma - 1)s_{11t}/\sigma < 1$, the combined derivative of the first two terms with respect to tariffs is always negative: $\frac{\partial}{\partial \tau_{21t}} [-\ln P_{1t} + n_{1t-1}\pi_{1t}] < 0$. Thus, without a revenue motive ($\rho = 0$), the lower-bound constraint on tariffs binds and the country chooses free trade. Households benefiting from tariff revenue ($\rho > 0$) can provide a motive for protection. Raising the tariff increases revenue collected on the remaining imports but also reduces import expenditure. The revenue term of the derivative is positive when the former effect dominates and negative when the contraction in imports dominates. The total derivative changes sign at most once, so the optimal discretionary tariff is unique. This discussion leads to the following proposition.

Proposition 2 (Markov perfect tariffs) *Suppose that $n_{2t-1} > 0$. Then the optimal discretionary tariff is unique and as follows:*

If $1 - \frac{\sigma-1}{\sigma} \frac{n_{1t-1}}{n_{2t-1}\theta^{1-\sigma} + n_{1t-1}} \leq \rho$ then

$$\tau_{21t} = \rho \frac{\sigma + \sigma(\sigma - 1)s_{11t}}{\sigma + (\sigma - 1)(\rho\sigma - 1)s_{11t}}. \quad (20)$$

If $1 - \frac{\sigma-1}{\sigma} \frac{n_{1t-1}}{n_{2t-1}\theta^{1-\sigma} + n_{1t-1}} \geq \rho$ then the lower bound on tariffs binds and $\tau_{21t} = 1$.

Proof. See Appendix B. ■

Note that since the domestic expenditure share s_{11t} depends on the tariff, the equation above is implicit.

3.3 Optimal policy under commitment

We now characterize some features of the optimal policy tariff path under commitment. We begin by stating the conditions that the time- t optimal tariff must satisfy, and highlight the role of dynamic considerations. We then discuss the optimal $t = 0$ tariff. Finally, we describe parameter conditions under which tariffs start low and increase over time, or vice versa.

The optimal tariff at time $t > 0$ under commitment The Lagrangian associated with the Ramsey problem (17) and the resulting first-order conditions are written out in Appendix A.2. Here we summarize the results.

The non-negativity constraint of the lower-bound multiplier and complementary slackness are

$$\lambda_t^{\tau_{LB}} \geq 0, \quad \lambda_t^{\tau_{LB}} (\tau_{21t} - 1) = 0.$$

When the time- t tariff is interior, it balances its effect on time- t welfare against the changes in both foreign and domestic entry induced in earlier periods based on the announcement of what this tariff will be at t . Combining the tariff, profit, and price-index first-order conditions yields³

$$0 = \underbrace{\frac{\partial}{\partial \tau_{21t}} \left[-\ln P_{1t} + n_{1t-1} \pi_{1t} + \rho(1 - \tau_{21t}^{-1}) s_{21t} \right]}_{\text{Marginal effect on period-}t \text{ welfare}} + \underbrace{\lambda_{1t-1}^v \frac{\partial \pi_{1t}}{\partial \tau_{21t}} + \lambda_{2t-1}^v \frac{\partial \pi_{2t}}{\partial \tau_{21t}}}_{\text{Marginal contribution through earlier entry}}. \quad (21)$$

Home operating profits satisfy $n_{1t-1} \pi_{1t} = (s_{11t} + s_{12t} / \tau_{12t}) / \sigma$. The derivative of export profits with respect to the current import tariff is zero at fixed stocks, as $s_{12t} = \frac{n_{1t-1} (\theta \tau_{12t})^{1-\sigma}}{n_{2t-1} + n_{1t-1} (\theta \tau_{12t})^{1-\sigma}}$. Home entry costs, $b^\chi \chi v_{1t}^{1+\chi} / (1 + \chi)$, likewise have no direct term in the tariff first-order condition. The effects of promised tariffs on firm values and entry are accounted for by the multiplier terms. The entry cost term's derivative with respect to τ_{21t} is therefore zero in (21).

The first term of equation (21) is familiar from the optimal policy under discretion and captures “static” incentives. The difference under commitment arises entirely from dynamic considerations and is reflected in the second term. At given stocks, a higher import tariff raises home operating profits and lowers foreign operating profits ($\frac{\partial \pi_{1t}}{\partial \tau_{21t}} > 0$ but $\frac{\partial \pi_{2t}}{\partial \tau_{21t}} < 0$). Thus, the $t - 1$ Lagrange multipliers on the firm values enter with opposite signs. A positive multiplier on the value of home firms supports protection: the higher profits promised to home firms encourage earlier entry that is socially beneficial on balance, as reflected in a positive λ_{1t-1}^v . This is the Venables delocation force: tariffs raise domestic profits and therefore the mass of domestic firms.

A positive multiplier on foreign firm value restrains protection: reducing foreign profits discourages foreign entry, whose value to home households is captured by λ_{2t-1}^v . This is a Chamley-

³Equation (21) follows from the tariff first-order condition after using the first-order conditions for operating profits and the home price index to eliminate their associated multipliers; see Appendix A.2. The derivatives in (21) are computed using the static equilibrium expressions for prices, profits, and expenditure shares. The terms involving λ_{it-1}^v arise because date- t profits enter the date- $t - 1$ firm-value constraints. Firm entry remains endogenous under commitment, and the effects of announced tariffs on earlier entry decisions are accounted for by these multiplier terms.

Judd-like force: announcing a future tax on foreign firms' exports to country 1 discourages their entry, eroding the time- t tax base and reducing tariff revenues, all else equal. Both contributions are valued in home welfare. When both multipliers are positive, the government weighs these opposing effects by the sensitivity of profits to the tariff.

Writing out and rearranging these terms yields:

$$\lambda_{1t-1}^v \frac{\partial \pi_{1t}}{\partial \tau_{21t}} + \lambda_{2t-1}^v \frac{\partial \pi_{2t}}{\partial \tau_{21t}} = \frac{s_{21t}}{\sigma \tau_{21t}^2} \left[(\sigma - 1) \tau_{21t} s_{11t} \frac{\lambda_{1t-1}^v}{n_{1t-1}} - [1 + (\sigma - 1) s_{11t}] \frac{\lambda_{2t-1}^v}{n_{2t-1}} \right]. \quad (22)$$

Plugging (19) and (22) into (21) and factoring out the common $\frac{s_{21t}}{\tau_{21t}}$ term leads to the following first-order condition when τ_{21t} is interior:

$$0 = 1 - \frac{\sigma - 1}{\sigma} s_{11t} + \rho(\sigma - 1)(1 - \tau_{21t}^{-1}) s_{11t} - \frac{\rho}{\tau_{21t}} - \frac{\sigma - 1}{\sigma} s_{11t} \frac{\lambda_{1t-1}^v}{n_{1t-1}} + \frac{1 + (\sigma - 1) s_{11t}}{\sigma \tau_{21t}} \frac{\lambda_{2t-1}^v}{n_{2t-1}}.$$

Rearranging, we obtain an implicit formula for the welfare-maximizing τ_{21t} :

$$\tau_{21t} = \frac{(\sigma \rho - \lambda_{2t-1}^v / n_{2t-1}) [1 + (\sigma - 1) s_{11t}]}{\sigma + (\sigma - 1) (\sigma \rho - 1 - \lambda_{1t-1}^v / n_{1t-1}) s_{11t}}. \quad (23)$$

Notice that this formula collapses to the optimal tariff under discretion when the two multipliers λ_{1t-1}^v and λ_{2t-1}^v are zero (equation 20). The focus of our discussion going forward is therefore on these two multipliers.

Promised protection raises home operating profits and lowers foreign operating profits, thereby influencing the entry that precedes it. The inherited value multipliers λ_{it-1}^v summarize country 1's gains from the entry induced by promised future profits. To further build intuition, we can relate them to the multipliers on the domestic and foreign firm stocks λ_{it}^n . Starting from $\lambda_{i,-1}^v = 0$, iteration of (A.5)–(A.6) gives, for $t \geq 1$,

$$\lambda_{1t-1}^v = \sum_{k=0}^{t-1} (1 - \delta)^{t-1-k} \chi b^\chi v_{1k}^{\chi-1} (\lambda_{1k}^n - v_{1k}), \quad (24)$$

$$\lambda_{2t-1}^v = \sum_{k=0}^{t-1} (1 - \delta)^{t-1-k} \chi b^\chi v_{2k}^{\chi-1} \lambda_{2k}^n. \quad (25)$$

These equations relate the multipliers on the value of firms at $t - 1$ to the discounted sum of the multipliers on the number of firms up to time $t - 1$. (See Appendix A.4 for the detailed derivation.)

To interpret the expressions above, consider the contribution of date- t profits to entry at an earlier date $k < t$. An additional unit of profit per firm at date t contributes $\frac{\partial v_{ik}}{\partial \pi_{it}} = \beta^{t-k} (1 - \delta)^{t-1-k}$ to firm value at date k . The discount factor reflects the delay before the profit is received, while the survival factor reflects the possibility that the firm exits before receiving it. The resulting increase in firm value encourages entry at date k , with the strength of this response given by $\frac{\partial (bv_{ik})^\chi}{\partial v_{ik}} = \chi b^\chi v_{ik}^{\chi-1}$, the derivative of entry with respect to firm value. Thus, the same increase in

promised profits induces more entry when entry is more responsive to firm value.

Whether this additional entry benefits home households depends on its social value and resource cost. An additional home firm at date k has social value λ_{1k}^n , which accounts for the consequences of the larger firm stock in the commitment problem. Creating that firm also uses domestic resources. Since the marginal entrant's entry cost equals its private firm value v_{1k} , the net social gain from additional home entry is $\lambda_{1k}^n - v_{1k}$. A promise that encourages home entry is therefore beneficial through this channel when the social value of the marginal domestic firms exceeds the resources required to create them. Higher entry is not automatically desirable merely because the entering firms are domestic.

For foreign entry, the corresponding value to home households is λ_{2k}^n . Foreign entry costs are borne abroad and therefore are not deducted from home welfare. Nevertheless, additional foreign entry need not always benefit home households: its value reflects the benefits of access to foreign varieties and rebated tariff revenue, and its effects on home profits. The multiplier λ_{2k}^n measures the balance of these effects from the home policymaker's perspective, rather than the welfare of foreign producers.

Each term in the two sums thus combines three elements: the contribution of promised date- t profits to earlier firm value, the responsiveness of entry to that value, and the net social value of the resulting entry. Summing over $k = 0, \dots, t-1$ accounts for all earlier entry decisions affected by those profits. The discount factors cancel when welfare effects are expressed in date- t units. This leaves the survival weights appearing in the formulas. The sums therefore run over earlier entry dates because they trace the investment decisions induced on the margin by the government's promise of time- t tariff. The social value assigned to each entry decision already incorporates its subsequent consequences.

All weights are strictly positive: they reflect the responsiveness of entry to firm value and the survival of firms from earlier cohorts. Consequently, if

$$\begin{aligned} \lambda_{1k}^n \geq v_{1k} \text{ for every } 0 \leq k < t &\implies \lambda_{1t-1}^v \geq 0, \\ \lambda_{2k}^n \geq 0 \text{ for every } 0 \leq k < t &\implies \lambda_{2t-1}^v \geq 0. \end{aligned}$$

A period $k \leq t-1$ with $\lambda_{1k}^n - v_{1k} > 0$ contributes positively to the firm-value multiplier λ_{1t-1}^v . A period with $\lambda_{2k}^n > 0$ contributes positively to λ_{2t-1}^v . Since λ_{ik}^n are multipliers on equality constraints, neither has a sign restriction of its own: the corresponding net social values can be negative. If net social gains change sign across periods, their weighted contributions determine the sign of the firm value multipliers λ_{1t-1}^v and λ_{2t-1}^v .

The inherited multipliers λ_{1t-1}^v and λ_{2t-1}^v thus summarize the net social gains from entry decisions over the earlier periods, weighted by entry responsiveness and firm survival. A net beneficial home entry path ($\lambda_{1t-1}^v > 0$) supports protection, whereas net beneficial foreign entry ($\lambda_{2t-1}^v > 0$) restrains it. When both are beneficial, the tariff condition weighs these cumulative social gains by the sensitivity of each country's profits to the current tariff. Which effect dominates

depends on both the value of additional firms and how strongly the promised tariff influences their entry.

Stationary long-run tariffs To see these forces more clearly, consider the optimal tariffs from the time period at which this economy converged to a stationary equilibrium. From this point forward, tariffs and all equilibrium objects are constant, and these stationary allocations are denoted by the absence of time subscripts. In a stationary allocation, the value and stock first-order conditions imply

$$\frac{\lambda_1^v}{n_1} = \chi \left(\frac{\lambda_1^n}{v_1} - 1 \right), \quad \frac{\lambda_2^v}{n_2} = \chi \frac{\lambda_2^n}{v_2}. \quad (26)$$

Substituting these identities into the interior optimal tariff formula (23) gives

$$\tau_{21} = \frac{(\sigma\rho - \chi\lambda_2^n/v_2) [1 + (\sigma - 1)s_{11}]}{\sigma + (\sigma - 1) [\sigma\rho - 1 - \chi(\lambda_1^n/v_1 - 1)] s_{11}}. \quad (27)$$

This expression applies to a positive finite stationary tariff when the denominator is nonzero. It remains implicit: the expenditure share and social values must be determined jointly with the stationary allocation. Appendix A.3.1 derives each step and explains why v_1 is the marginal, rather than the average, entry cost.

Protection raises home profits and lowers foreign profits. If home entry is socially worthwhile, $\lambda_1^n > v_1$, the entry induced by promised protection contributes positively to the marginal benefit of a tariff, and higher λ_1^n/v_1 raises the optimal tariff, all else equal. If foreign entry benefits home households, $\lambda_2^n > 0$, higher value of foreign firms to domestic households lowers the optimal tariff, all else equal. These two effects thus oppose each other when both λ_i^v 's are positive. The entry elasticity χ scales these contributions at given normalized social values; those values also adjust when χ changes.

The optimal tariff at $t = 0$ We now characterize the optimal tariff at date 0. The first-period tariff problem is qualitatively different because it involves no dynamic considerations. The present tariffs affect no one's entry decision, either at home or abroad. The $t = 0$ tariff therefore solves the static optimization problem

$$\max_{\tau_{21,0} \in [1, \infty)} \left[-\ln P_{1,0} + \frac{s_{11,0}}{\sigma} + \rho(1 - \tau_{21,0}^{-1})s_{21,0} \right]. \quad (28)$$

In the dynamic optimization with commitment, the government still chooses the entire future tariff path. The omitted export profits and entry costs thus remain in the full dynamic objective. The path of tariffs for $t \geq 1$ does not affect the first-order conditions for the $t = 0$ tariff: for every feasible continuation, the $t = 0$ tariff that solves (28) is optimal given the initial/inherited stocks $n_{1,-1}$ and $n_{2,-1}$. Getting ahead, this is why the first-period optimal tariff under commitment coincides with the discretionary current choice conditional on the pre-determined stocks of firms.

The first-order condition is

$$\begin{aligned} & \frac{\partial}{\partial \tau_{21,0}} \left[-\ln P_{1,0} + \frac{s_{11,0}}{\sigma} + \rho(1 - \tau_{21,0}^{-1})s_{21,0} \right] \\ &= \frac{s_{21,0}}{\tau_{21,0}} \left[-1 + \frac{\sigma-1}{\sigma}s_{11,0} - \rho(\sigma-1)(1 - \tau_{21,0}^{-1})s_{11,0} + \frac{\rho}{\tau_{21,0}} \right]. \end{aligned} \quad (29)$$

Dividing by the positive factor $s_{21,0}/\tau_{21,0}$ makes the economic tradeoff particularly transparent:

$$\underbrace{-1}_{\text{Consumer prices}} + \underbrace{\frac{\sigma-1}{\sigma}s_{11,0}}_{\text{Home profits}} + \underbrace{\frac{\rho}{\tau_{21,0}} [1 - (\sigma-1)(\tau_{21,0} - 1)s_{11,0}]}_{\text{Rebated revenue}}.$$

The normalized consumer loss is -1 . Current home-profit gains offset only part of that loss, because $(\sigma-1)s_{11,0}/\sigma < 1$. Thus, at fixed stocks of firms, a tariff always lowers welfare through the combined impact of consumer prices and home profits. Rebated revenue can provide a reason to impose a tariff. As the tariff rises, erosion of the import base makes that revenue motive weaker – a Laffer curve.

The optimal $t = 0$ tariff, when it is interior, sets (29) equal to zero. Rearranging the first-order condition gives the implicit formula:

$$\tau_{21,0} = \frac{\sigma\rho[1 + (\sigma-1)s_{11,0}]}{\sigma + (\sigma-1)(\sigma\rho-1)s_{11,0}}, \quad (30)$$

where $s_{11,0}$ is evaluated at the same tariff. Its denominator is strictly positive, since it is at least $\sigma - (\sigma-1)s_{11,0} > 0$. The single-crossing argument in the proof of Proposition 3 (Appendix B) makes this implicit equation sufficient at an interior optimum.⁴

At the same time, for low enough ρ the optimization problem (28) may not have an interior solution. That cutoff ρ is found by setting (29) to 0 while at the same time setting $\tau_{21,0} = 1$. This discussion leads to the following proposition.

Proposition 3 (Characterization of the $t = 0$ tariff) *Let $\sigma > 1$, $0 \leq \rho \leq 1$, $1 \leq \theta < \infty$, and $n_{1,-1}, n_{2,-1} > 0$. Take the foreign tariff path as given. The period-0 problem has a unique global optimum. Whenever the full commitment maximum is attained, every optimal commitment plan must begin with this same $t = 0$ tariff. The optimal $t = 0$ tariff has the following form:*

(i) Zero tariff. The first-period optimum is $\tau_{21,0} = 1$ if and only if

$$\rho \leq 1 - \frac{\sigma-1}{\sigma} \frac{n_{1,-1}}{n_{1,-1} + n_{2,-1}\theta^{1-\sigma}}. \quad (31)$$

⁴At an interior solution the second derivative can also be checked directly. Factor the first derivative as the positive prefactor $s_{21t}[1 - (\sigma-1)(\tau_{21t}-1)s_{11t}]/\tau_{21t}^2$ times the difference between ρ and $\frac{\tau_{21t}[1 - (\sigma-1)s_{11t}/\sigma]}{1 - (\sigma-1)(\tau_{21t}-1)s_{11t}}$. The latter difference vanishes at the root, so differentiating gives $\frac{\partial^2}{\partial \tau_{21t}^2} \left[-\ln P_{1t} + \frac{s_{11t}}{\sigma} + \rho(1 - \tau_{21t}^{-1})s_{21t} \right] = -\frac{s_{21t}}{\tau_{21t}^2} \frac{1 + (\sigma-1)^2(\tau_{21t}-1)s_{11t}s_{21t}}{1 - (\sigma-1)(\tau_{21t}-1)s_{11t}} < 0$. This inequality is evaluated at an interior first-period or reset solution with inherited stocks fixed. It does not verify the second-order conditions for the entire commitment path.

(ii) Interior tariff. *The first-period optimum is given by (30) if and only if*

$$\rho > 1 - \frac{\sigma - 1}{\sigma} \frac{n_{1,-1}}{n_{1,-1} + n_{2,-1}\theta^{1-\sigma}}.$$

Proof. See Appendix B. ■

The dynamic path of tariffs under commitment The preceding discussion highlights that in a dynamic environment, the path of optimal tariffs is affected by the impact of those tariffs on entry at home and abroad. The current tariff does not directly enter the contemporaneous entry condition, but a tariff announced for a future date affects entry before it takes effect. Hence, the time- $t > 0$ tariff will often differ from the $t = 0$ tariff. This is immediate from comparing the optimal tariff formulas at $t = 0$ in (30) to the formula for $t > 0$ in (23). The difference arises from the dynamic considerations embodied in the multiplier terms $\lambda_{2t-1}^v/n_{2t-1}$ and $\lambda_{1t-1}^v/n_{1t-1}$. As discussed above, these terms reflect the value of the foreign and domestic entry, respectively, that is induced prior to time t by the tariff announced for time t .

Here, we use the difference between (30) and (23) to build intuition for when the optimal path of tariffs goes from low at $t = 0$ to high at future dates, and when the opposite happens. Combining (21) and (22), the first-order condition for an interior optimal time- t tariff becomes:

$$\begin{aligned} & \frac{\partial}{\partial \tau_{21t}} \left[-\ln P_{1t} + n_{1t-1}\pi_{1t} + \rho(1 - \tau_{21t}^{-1})s_{21t} \right] \\ & = -\frac{s_{21t}}{\sigma\tau_{21t}^2} \left[(\sigma - 1)\tau_{21t}s_{11t}\frac{\lambda_{1t-1}^v}{n_{1t-1}} - [1 + (\sigma - 1)s_{11t}]\frac{\lambda_{2t-1}^v}{n_{2t-1}} \right]. \end{aligned} \quad (32)$$

The first line is the derivative of the $t = 0$ problem under stocks n_{1t-1} and n_{2t-1} . Since $s_{21t}/(\sigma\tau_{21t}^2) > 0$, the dynamic considerations reflected in the second line lead to higher than static tariffs if and only if the term in brackets is positive:

$$(\sigma - 1)\tau_{21t}s_{11t}\frac{\lambda_{1t-1}^v}{n_{1t-1}} > [1 + (\sigma - 1)s_{11t}]\frac{\lambda_{2t-1}^v}{n_{2t-1}}, \quad (33)$$

and vice versa. By (24), the left side of the inequality is the marginal value of cumulative domestic entry up to $t - 1$ triggered by the announced tariff at t . The right-hand side is the marginal value to the country 1 consumers of the foreign entry triggered by the time- t tariff announcement. When the LHS is greater than the RHS, the Venables motive dominates: an import tariff engendering domestic entry prevails over the negative impact of the tariff on foreign entry. Thus, the policymaker wants to announce a high future tariff, for precisely the reasons analyzed by Venables (1987) and Ossa (2011). When the opposite is true, the policymaker wants to announce that tariffs will fall in the future. Intuitively, this happens when high tariffs discourage entry abroad, eroding the tax base of the tariff. This tax base erosion does not operate within a period, as the mass of foreign firms is predetermined. Thus, in this case the policymaker wants to announce that, while tariffs

are high today, they will be lower in the future.

While it builds intuition, this discussion is not quite a true comparison between $t = 0$ tariffs and time- t tariffs, because it is not necessarily the case that the exogenous initial mass of firms $n_{1,-1}$ equals the mass of firms present at t , n_{1t-1} . More precisely, it can be viewed as highlighting the difference between the time- t optimal tariff as pre-announced at $t = 0$, and the time- t optimal tariff for a policymaker that decides to reoptimize at t . The comparison (33) then tells us whether, having arrived at t , the policymaker wants to reoptimize the tariffs down or up from the preannounced level. When the Venables force is dominant, the policymaker is tempted to lower tariffs relative to the pre-announced ones, and vice versa. Appendix A.4 derives this reoptimization result. The next subsection compares the initial and stationary tariffs along the commitment path.

3.4 Ranking initial and stationary tariffs: sufficient conditions

The above results, in particular (23), (27), and (30) provide an implicit characterization of the time path of tariffs, expressed in terms of endogenous shares and Lagrange multipliers. We also fully characterize the set of primitive restrictions under which the $t = 0$ tariff is zero. To further build intuition on how the commitment tariff path depends on parameter values, we now state a set of sufficient conditions for different values of both $t = 0$ and stationary tariffs. We assume, as above, that the optimal commitment allocation exists and converges: firm stocks converge to positive finite limits and the current-value Lagrange multipliers to finite limits, while the tariff converges to a stationary tariff τ_{21} that can be infinite. Entry remains strictly unsaturated, $(bv_{it})^\chi < 1$, and the first-order and transversality conditions hold along the convergent path. Maintain $\sigma > 1$, $1 < \theta < \infty$, $0 < \chi < \infty$, $\rho \in [0, 1]$, and $\tau_{12t} = 1$, together with the restrictions on β, δ, b, p adopted throughout. Each economy starts from its symmetric free-trade steady state, so $n_{1,-1} = n_{2,-1} > 0$. This initial condition describes the firm stocks inherited before policy is announced; it does not restrict the $t = 0$ policy to free trade. We state three theoretical results. Lemmas 6 and 7 provide parameter restrictions under which the initial and the stationary tariffs, respectively, are zero or positive. Proposition 4 then puts these results together. It describes regions of the parameter space in which the commitment path starts with low tariffs but converges to higher ones in steady state, and vice versa.

Lemma 6 (The $t = 0$ tariff) *The $t = 0$ tariff is uniquely determined. It satisfies*

$$\tau_{21,0} = 1 \iff \rho \leq \frac{1/\sigma + \theta^{1-\sigma}}{1 + \theta^{1-\sigma}}. \quad (34)$$

Under the strict reverse inequality, the initial tariff is positive and finite, with

$$1 < \tau_{21,0} < \frac{\sigma^2 \rho}{1 + \sigma(\sigma - 1)\rho}. \quad (35)$$

Conditional on the inherited firm stocks $n_{i,-1}$, the $t = 0$ tariff is independent of the entry elasticity χ .

Proof. See Appendix B. ■

The $t = 0$ tariff raises consumer prices, shifts expenditure toward home producers, and returns a fraction ρ of tariff revenue to households. The consumer loss exceeds the gain in home operating profits, so rebated revenue must be sufficiently valuable to justify protection at $t = 0$. With $\rho = 0$, the initial tariff is therefore zero. A higher θ lowers the import expenditure share at free trade and increases the home-profit offset relative to the consumer loss; the threshold for ρ in (34) consequently falls. The entry elasticity does not affect this first choice because the initial firms already exist. Although χ affects the level of the initial symmetric stocks, it does not affect the expenditure shares that determine this tariff.

Lemma 7 (Long-run protection) *Under the maintained existence, convergence, and regularity assumptions, the stationary tariff is finite. The following conditions are sufficient to determine whether it is zero or positive.*

- (i) Zero protection. Fix σ, χ, ρ such that $\rho < \frac{1}{\sigma(1+\chi)}$. For all sufficiently large finite θ , the stationary tariff is $\tau_{21} = 1$.
- (ii) Positive protection. The stationary tariff satisfies $1 < \tau_{21} < \infty$ if either
 - (a) ρ is sufficiently large: $\sigma \geq 2$, $\rho \geq \frac{\sigma}{2(\sigma-1)}$ or
 - (b) χ is sufficiently large: $\chi \geq \frac{(1+\theta^{1-\sigma})^2}{2\theta^{1-\sigma}(1-\theta^{1-\sigma})}$.

The condition (b) applies to every $\rho \in [0, 1]$ and $\sigma > 1$.

Proof. See Appendix B. ■

Future tariffs affect the entry that precedes them as well as the welfare in the period they are applied. When little revenue is returned to households and the gains from inducing entry are sufficiently weak, these additional benefits cannot compensate for the contemporaneous consumer loss. Part (i) establishes this conclusion when trade costs are high enough to limit the feedback between the two markets. Its restriction on ρ becomes more demanding as entry becomes more responsive. Part (ii) gives two reasons for positive long-run protection: sufficiently valuable fiscal receipts, or an entry response strong enough to make future protection worthwhile even when tax revenue is not highly valued. The latter is precisely the Venables delocation force, which operates even without any fiscal revenue benefit of tariffs. These are sufficient conditions on parameter regions, not an exhaustive set nor a claim that tariffs increase with χ at every parameter value.

The optimal stationary tariffs are finite. Near the prohibitive tariff, foreign profits from the home market are already extremely small, so a further tariff increase has too little effect on foreign entry to offset the loss in import consumption. Positive rebates provide an additional benefit from reopening trade. The limit of infinite tariffs is excluded under the maintained regularity conditions, including when $\rho = 0$.

The following proposition describes (a nonexhaustive set of) parameter configurations under which long-run tariffs exceed or fall below their initial levels along the commitment tariff path.

Proposition 4 (Initial vs. long-run protection)

(i) Higher long-run protection. The economy features $1 = \tau_{21,0} < \tau_{21} < \infty$ if entry is sufficiently elastic $\chi \geq \frac{(1+\theta^{1-\sigma})^2}{2\theta^{1-\sigma}(1-\theta^{1-\sigma})}$ and rebates are sufficiently low $\rho \leq \frac{1/\sigma + \theta^{1-\sigma}}{1+\theta^{1-\sigma}}$.⁵

(ii) Lower long-run protection. When trade costs are sufficiently low, entry is sufficiently inelastic (χ sufficiently low), and $\rho \in \left(\frac{1/\sigma + \theta^{1-\sigma}}{1+\theta^{1-\sigma}}, \frac{\sigma+1}{2\sigma} + \frac{1}{4\sigma^3}\right)$, then $1 \leq \tau_{21} < \tau_{21,0}$.⁶

Proof. See Appendix B. ■

The higher future protection regions arise because future protection can encourage entry at home. This impact on entry adds a benefit that the $t = 0$ tariff cannot provide. When rebates do not justify protection at $t = 0$ (low ρ), sufficiently responsive entry can nevertheless support positive long-run tariffs. When both tariffs are positive, sufficiently high trade costs make the benefits of supplying varieties domestically important enough to raise long-run protection above its initial level. This domestic-entry channel is related to the Venables delocation mechanism. The lower future protection regions reflect the cost of discouraging valuable entry through taxes on future export profits. With low trade costs, foreign varieties are an important source of consumption benefits for home households. The policymaker with the ability to commit can encourage their entry by promising lower future tariffs. This is the investment-incentive logic associated with front-loaded capital taxation: existing firms can be taxed without changing the entry that created them, whereas future taxes affect earlier investment decisions.

The entry elasticity strengthens the response to promised future profits in both countries. It can magnify the gain from encouraging home entry or the cost of discouraging beneficial foreign entry; its effect on the direction of tariff adjustment depends on their relative importance. Rebates affect the current fiscal incentive and the value of the import tax base, while trade costs affect the relative value of domestic and imported varieties.

3.5 Graphical illustrations: delocation and the timing of protection

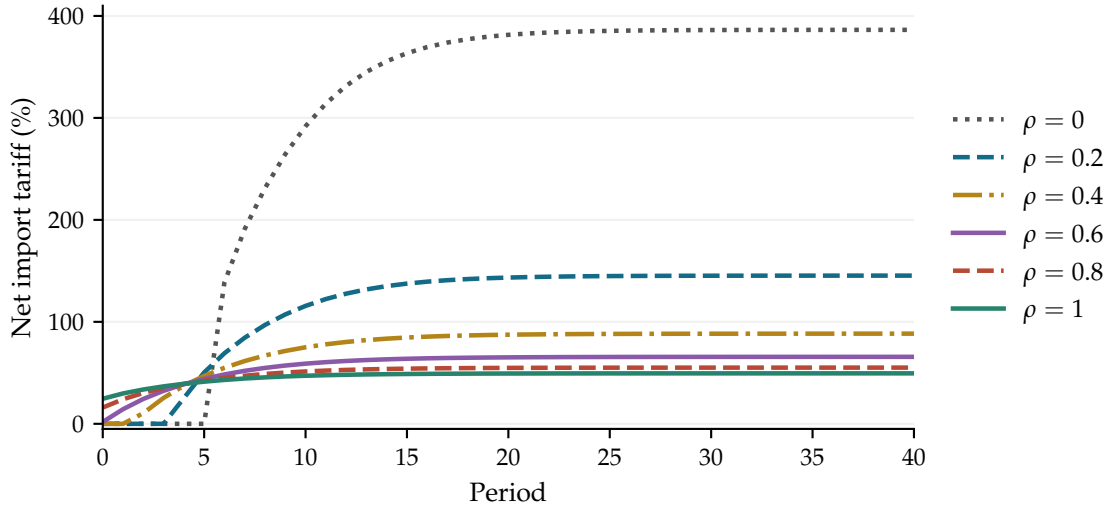
We now provide numerical illustrations of the optimal tariff dynamics under commitment. We focus on highlighting how the parameter values govern the strength of the key competing mechanisms: entry incentives and tax revenue. Throughout, we set $\sigma = 3$, $\beta = 0.97$, $\delta = 0.25$, $b = p = 1$, and $\tau_{12t} = 1$. The world economy at $t = 0$ inherits the symmetric free-trade steady state. The vertical axes report net import tariffs in percent, $100 \cdot (\tau_{21t} - 1)$. The equilibrium and tariff-path computations are described in Appendix C.

Delocation, protection, and tariff revenue Figure 2 fixes $\theta = 1.3$ and $\chi = 3$ and varies the rebate share ρ . Under these values of the parameters, the Venables delocation force dominates the dynamics: promised future protection encourages domestic entry, providing a motive for long-run

⁵Alternatively, for any $\chi > 0$ and $\rho \in (1/\sigma, 1]$, if the economy is sufficiently closed (θ sufficiently large; the required θ rises as χ falls), we also get $1 < \tau_{21,0} < \tau_{21} < \infty$.

⁶The lower bound in (ii) can be attained: Appendix B.9.4 gives explicit, if narrow, conditions under which initial protection is followed by long-run free trade, $1 = \tau_{21} < \tau_{21,0}$.

Figure 2: Delocation, protection, and tariff revenue



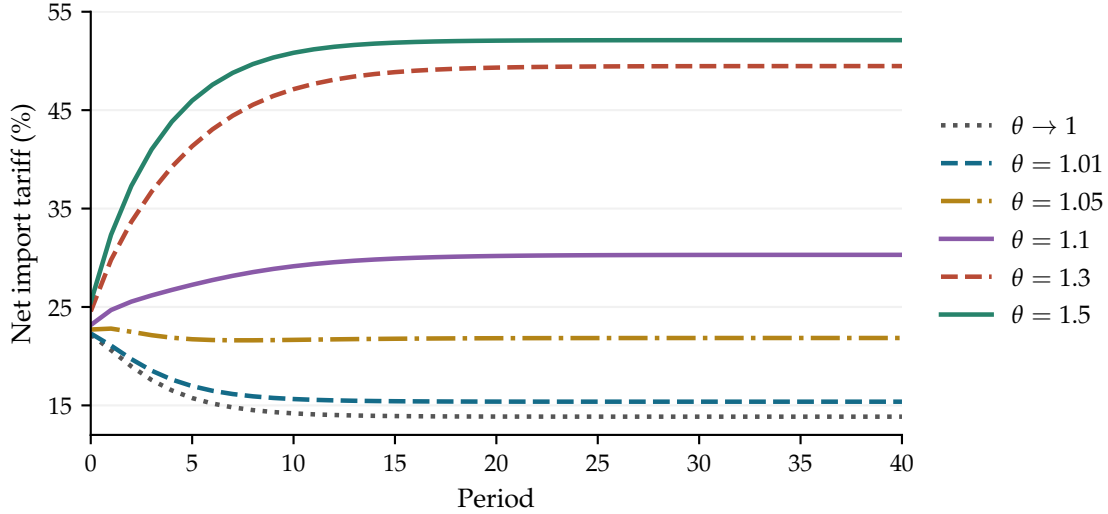
Notes: The figure fixes $\sigma = 3$, $\theta = 1.3$, and $\chi = 3$, and compares $\rho = 0, 0.2, 0.4, 0.6, 0.8, 1$. The other model parameters are as detailed in the main text.

protection even when tariff revenue is not rebated ($\rho = 0$). However, the initial tariff cannot induce entry that has already occurred, so this benefit arises through the promise of future protection. At $\rho = 0$, the optimal path begins at free trade and introduces protection at date $t = 6$, eventually reaching a net tariff of 386%. The long-run tariff essentially recovers the Venables outcome: the country imposes prohibitive tariffs. However, in a dynamic setting the prohibitive tariffs do not happen right away.

As ρ increases, the government places greater value on tariff revenue. High tariffs reduce imports and discourage foreign entry, eroding the future import tax base. Preserving this base becomes more valuable when a larger share of revenue is returned to households, adding to the cost of future protection. In the illustrated economy, high ρ brings protection forward while lowering the long-run tariff. At $\rho = 1$ (all tariff revenue is valuable), the initial net tariff is 25% and the long-run rate is 49%. The fiscal incentive therefore affects initial and future protection differently: it makes taxing $t = 0$ imports more attractive while increasing the value of sustaining imports over time.

Trade costs and the timing of protection Figure 3 fixes $\chi = 3$ and $\rho = 1$ and varies the iceberg trade cost. The model contains two opposing forces. The Venables delocation force supports future protection because it encourages domestic entry. The Chamley–Judd force favors taxing existing firms today while promising lower future taxes to preserve entry incentives. More precisely, import tariffs tax the sales that foreign entrants earn from serving the home market. Existing firms can be taxed today without changing the entry that created them, whereas promised future tariffs

Figure 3: Trade costs and the timing of protection: Venables vs. Chamley–Judd



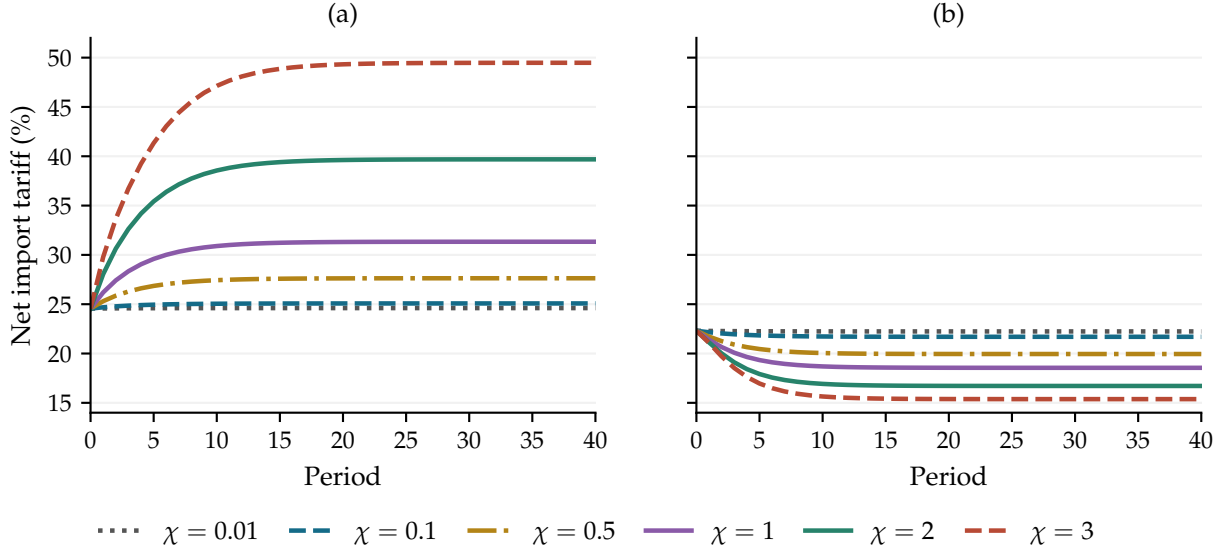
Notes: The figure fixes $\sigma = 3$, $\chi = 3$, and $\rho = 1$, and compares $\theta \rightarrow 1$, 1.01, 1.05, 1.1, 1.3, and 1.5. The limiting label denotes the numerical approximation $\theta = 1 + 10^{-10}$. The other model parameters are as detailed in the main text.

affect earlier entry decisions.

The trade cost θ governs the relative strength of these forces in the figure. When trade costs are high, hosting varieties domestically is particularly valuable, and tariffs rise over time. When trade costs are low, access to foreign varieties is more valuable, and tariffs decline over time. Both forces coexist, and the declining paths retain positive long-run tariffs. Intermediate trade costs can produce nonmonotone adjustment, as the path at $\theta = 1.05$ illustrates. The curve labeled $\theta \rightarrow 1$ represents trade with negligible iceberg costs. When there are no trade costs, the Venables delocation force is at its weakest, because the cost savings to consumers from relocating a firm from abroad to home are absent. This is the most clear-cut case where the Chamley–Judd force dominates, and long-run tariffs converge to a lower value than the $t = 0$ tariffs.

Entry elasticity amplifies both forces Figure 4 holds $\rho = 1$ fixed and varies χ in two economies: the Venables economy in panel (a) and Chamley–Judd in panel (b). A larger χ makes entry more responsive to promised future profits. In the Venables world ($\theta = 1.3$), a stronger entry response increases the delocation benefit of future protection, producing a larger increase in long-run tariffs, all else equal. In the Chamley–Judd world ($\theta = 1.01$), more elastic entry raises the cost of taxing future returns, producing a larger decline in tariffs. The initial tariff remains unchanged within each panel because the initial stocks are symmetric across the two countries; the differences emerge only through the dynamic incentives created by future policy. More elastic entry can thus amplify either rising or declining protection, depending on which entry effect dominates.

Figure 4: Entry elasticity and the incentives for future protection or future tax relief



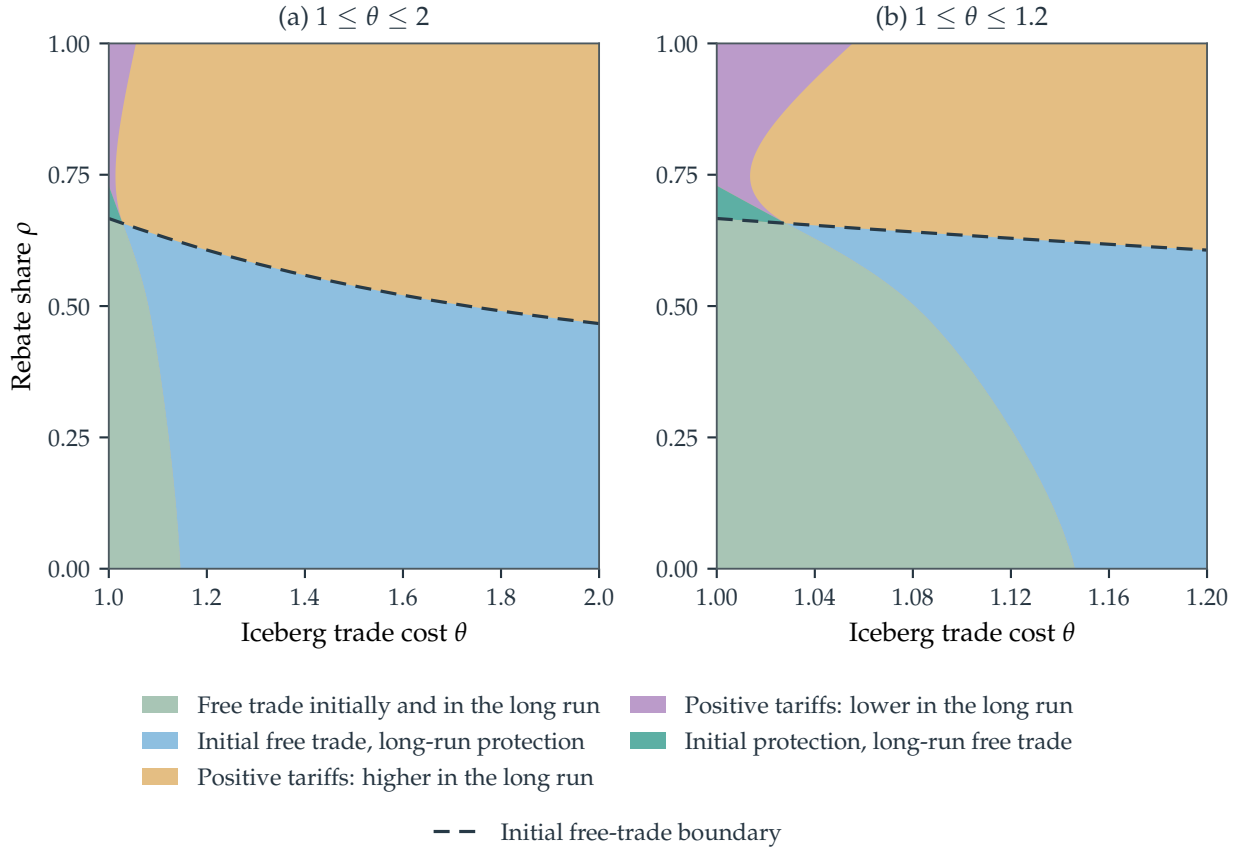
Notes: Both panels fix $\sigma = 3$ and $\rho = 1$ and compare $\chi = 0.01, 0.1, 0.5, 1, 2, 3$. Panel (a) sets $\theta = 1.3$; panel (b) sets $\theta = 1.01$. The panels use a common vertical scale. The other model parameters are as detailed in the main text.

Trade costs, rebates, and the endpoints of protection Panel (a) of Figure 5 fixes $\sigma = 3$ and $\chi = 3$ and compares initial and long-run tariffs over $1 \leq \theta \leq 2$ and $0 \leq \rho \leq 1$. Each economy starts from its symmetric private free-trade steady state, with $\beta = 0.97$, $\delta = 0.25$, $b = p = 1$, and $\tau_{12t} = 1$. The dashed curve, $\rho = (1/\sigma + \theta^{1-\sigma})/(1 + \theta^{1-\sigma})$, separates $t = 0$ free trade from $t = 0$ protection. We compute the optimal commitment policy numerically by maximizing discounted home welfare, including the transition. The figure compares the initial tariff with the long-run tariff under this policy. Adjustment between them need not be monotone. Panel (b) zooms in on the area of low trade costs ($1 \leq \theta \leq 1.2$) to make it easier to see.

The Venables delocation force dominates in the long run under higher trade costs (light blue and light orange areas). When trade costs are high, domestic entry at the expense of foreign entry avoids a greater loss of goods in transit. Promised protection can encourage domestic entry, making future protection attractive even when the optimal initial tariff is zero. The long-run tariff exceeds the initial tariff when this delocation benefit dominates. When trade costs and tariff revenue valuations are low, free trade is found to be optimal in both the short and the long run (light green). By contrast, when ρ is high but θ is low, the Chamley-Judd force dominates and tariffs fall from $t = 0$ to the steady state (purple). There is a small area (teal) with initial protection but zero steady-state tariffs.⁷ A larger ρ makes taxing the existing import base more attractive. Its impli-

⁷The presence of light green and teal areas in which trade is free in the long run under low θ may appear to contradict Lemma 7(i), that proves existence of a long-run free trade region for θ sufficiently high. There is no contradiction, in the

Figure 5: Trade costs and rebates: ranking initial and long-run protection



Notes: Both panels fix $\sigma = 3$ and $\chi = 3$. Panel (a) covers $1 \leq \theta \leq 2$; panel (b) enlarges $1 \leq \theta \leq 1.2$. Equality on the dashed curve applies to initial free trade. Regional boundaries are located using the tariff equations and, where steady states coexist, complete-path welfare comparisons. Stationary identification retains the maintained assumptions on the optimal path, and numerical welfare comparisons do not certify global optimality. The other model parameters are as detailed in the main text.

cations for future protection depend on entry. When trade costs are low (θ close to 1), imported varieties reach home consumers cheaply, making foreign entry especially valuable. Promising lower future tariffs preserves this entry and the future import tax base.

4 Conclusion

The sharp post-2016 tariff regime change has generated renewed interest in the unilateral optimal tariff setting under full policymaker discretion. While complete discretion implies freedom from

sense that Lemma 7(i) provides a sufficient condition, and does not rule out existence of other parameter configurations under which trade is free in the long run. The region of long-run free trade in Figure 5 is thus not ruled out by the lemma, but by the same token we cannot ensure its existence under all parameter configurations. It is possible that when other parameters (not ρ and θ) take on different values, this region disappears.

multilateral constraints imposed by trade agreements like the WTO, full discretion may also mean lack of commitment intertemporally. The experience, especially from 2025 onwards, is indeed suggestive of a policymaker unable to stick to its own announcements. In this paper, we argue that inability to stick to high tariffs is a natural outcome of a dynamic optimal tariff setting problem in which (i) optimal tariffs are driven by the Venables (1987) delocation motive, as verbalized by Donald Trump on many occasions; and (ii) the policymaker lacks the ability to commit. Since delocation takes time, but the higher prices are immediate, a policymaker that lacks the resolve to bear the short-run pain of higher prices will not succeed in imposing the optimal tariffs that would ultimately have induced long-run delocation and thereby a manufacturing renaissance.

We stress that this paper does not advocate high tariffs. Importantly, we analyze the unilateral optimal tariff problem, abstracting from retaliation or trade negotiations. Thus, we do not characterize the welfare under a trade war, nor compare it to welfare under the optimal cooperative tariffs. Our model also abstracts from many important real-world features, including less distortionary instruments for raising government revenue. Our objective is instead to clarify the dynamic incentives that shape optimal unilateral policy and to show that these incentives generate a time-inconsistency problem, which can rationalize the policy reversals of the Trump administration.

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ONLINE APPENDIX
(NOT FOR PUBLICATION)

A Lagrangian and common Ramsey derivations

In this section, we provide detailed derivations for Section 3.

A.1 From the model to the Ramsey problem

This subsection proves Lemma 4. The initial firm stocks and the foreign tariff path are given. All allocations considered satisfy the restrictions in the lemma. We first derive household welfare and the equilibrium conditions that appear as constraints in (17). We then show how to solve for the remaining equilibrium objects.

Step 1. Household choices and factor prices. Positive production of the homogeneous good implies $w_{jt} = 1$. Household optimality and bond-market clearing give

$$C_{jt}^m = P_{jt}^{-1}, \quad C_{jt}^h = L_j + \Pi_{jt} + \rho R_{jt} - 1, \quad B_{jt} = 0, \quad \frac{1}{1 + r_{jt}} = \beta.$$

Manufacturing expenditure is therefore $P_{jt}C_{jt}^m = 1$. For home households, flow welfare becomes

$$\ln C_{1t}^m + C_{1t}^h = -\ln P_{1t} + L_1 + \Pi_{1t} + \rho R_{1t} - 1.$$

Labor income is captured by the constant L_1 , as the wage is pinned down by the homogeneous good.

Step 2. Manufacturing prices and operating profits. The pricing rule in Section 2 gives $p_{ijt}^x = p\theta_{ij}$ and $p_{ijt}^c = \tau_{ijt}p\theta_{ij}$, where $\theta_{ii} = \tau_{iit} = 1$ and $\theta_{ij} = \theta$ for $i \neq j$. Substituting unit manufacturing expenditure into CES demand gives

$$q_{ijt} = P_{jt}^{\sigma-1} (\tau_{ijt}p\theta_{ij})^{-\sigma}.$$

The CES price index is consequently

$$P_{jt}^{1-\sigma} = \sum_{i=1}^2 n_{it-1} (\tau_{ijt}p\theta_{ij})^{1-\sigma},$$

which yields the two price-index constraints in (17). Since $p = \sigma c / (\sigma - 1)$, the profit margin on a shipment is

$$p\theta_{ij} - c\theta_{ij} = \frac{p\theta_{ij}}{\sigma}.$$

Operating profits per firm are therefore

$$\begin{aligned} \pi_{it} &= \sum_{j=1}^2 \frac{p\theta_{ij}}{\sigma} P_{jt}^{\sigma-1} (\tau_{ijt}p\theta_{ij})^{-\sigma} \\ &= \frac{p^{1-\sigma}}{\sigma} \sum_{j=1}^2 \theta_{ij}^{1-\sigma} \tau_{ijt}^{-\sigma} P_{jt}^{\sigma-1}. \end{aligned}$$

Separating domestic and export sales gives the two profit constraints in (17). The tariff wedge reduces producer revenue relative to consumer expenditure; this accounts for the exponent $-\sigma$ in export profits.

Step 3. Entry, firm stocks, and firm values. With $w_{it} = 1$, the marginal entrant has a sunk cost $\bar{\xi}_{it} = v_{it}$. Interior entry places this cutoff inside the support of the entry-cost distribution. The mass of entrants is $(bv_{it})^\chi$, so

$$n_{it} = (1 - \delta)n_{it-1} + (bv_{it})^\chi.$$

Their aggregate labor cost is

$$\int_0^{v_{it}} \xi \chi b^\chi \xi^{\chi-1} d\xi = b^\chi \frac{\chi}{\chi+1} v_{it}^{\chi+1}.$$

Home household income from firms is thus operating profits less the costs paid by current entrants:

$$\Pi_{1t} = n_{1t-1} \pi_{1t} - b^\chi \frac{\chi}{\chi+1} v_{1t}^{\chi+1}.$$

The one-period time-to-build and the household discount factor give

$$v_{it} = \beta [\pi_{it+1} + (1 - \delta)v_{it+1}].$$

Note that

$$0 \leq [\beta(1 - \delta)]^k v_{it+k} < \frac{[\beta(1 - \delta)]^k}{b} \rightarrow 0,$$

so iteration yields

$$v_{it} = \sum_{k=1}^{\infty} \beta^k (1 - \delta)^{k-1} \pi_{it+k}.$$

Step 4. Rebated revenue and household welfare. With unit manufacturing expenditure, the import expenditure share is

$$s_{21t} = P_{1t}^{\sigma-1} n_{2t-1} (\theta \tau_{21t} p)^{1-\sigma} = 1 - P_{1t}^{\sigma-1} n_{1t-1} p^{1-\sigma}.$$

The revenue collected on imports equals

$$\begin{aligned} R_{1t} &= (\tau_{21t} - 1) n_{2t-1} p \theta q_{21t} \\ &= (1 - \tau_{21t}^{-1}) s_{21t} \\ &= (1 - \tau_{21t}^{-1}) \left[1 - P_{1t}^{\sigma-1} n_{1t-1} p^{1-\sigma} \right]. \end{aligned}$$

Substituting this revenue and the net profits from Step 3 into the welfare expression in Step 1 gives

$$\begin{aligned} \ln C_{1t}^m + C_{1t}^h &= -\ln P_{1t} + n_{1t-1} \pi_{1t} - b^\chi \frac{\chi}{\chi+1} v_{1t}^{\chi+1} \\ &\quad + \rho(1 - \tau_{21t}^{-1}) \left[1 - P_{1t}^{\sigma-1} n_{1t-1} p^{1-\sigma} \right] + L_1 - 1. \end{aligned}$$

Discounting and summing gives (18), since $\sum_{t=0}^{\infty} \beta^t (L_1 - 1) = (L_1 - 1)/(1 - \beta)$. The rebate rate enters household welfare only through the fraction of revenue returned to households.

Step 5. The full equilibrium. Steps 1–4 show that the equilibrium satisfies the constraints in (17). Take prices, profits, firm stocks, and firm values satisfying these constraints, the given initial stocks, the tariff bounds, and the maintained feasibility restrictions. Set wages to one and interest rates as in Step 1; solve firm prices and quantities from Step 2, and entrant cutoffs from Step 3. These choices satisfy household optimality conditions, profit maximization, entry decisions, and the law of motion of firm stocks.

For each country, recover homogeneous-good consumption from the budget constraint and assign residual labor to homogeneous-good production. Manufacturing labor use is $(\sigma - 1)n_{jt-1}\pi_{jt}$, given that producer revenue is σ times operating profits and the wage is one. Thus,

$$\begin{aligned} C_{jt}^h &= L_j + n_{jt-1}\pi_{jt} - b^\chi \frac{\chi}{\chi + 1} v_{jt}^{\chi+1} + \rho R_{jt} - 1, \\ Q_{jt}^h &= l_{jt}^h = L_j - (\sigma - 1)n_{jt-1}\pi_{jt} - b^\chi \frac{\chi}{\chi + 1} v_{jt}^{\chi+1}. \end{aligned}$$

Note that the two countries' manufacturing expenditure totals two. Producer revenue equals this expenditure less total tariff revenue, so

$$\sigma \sum_{j=1}^2 n_{jt-1}\pi_{jt} = 2 - R_{1t} - R_{2t}.$$

It follows that

$$\begin{aligned} \sum_{j=1}^2 \left[C_{jt}^h + (1 - \rho)R_{jt} \right] - \sum_{j=1}^2 Q_{jt}^h &= \sigma \sum_{j=1}^2 n_{jt-1}\pi_{jt} - 2 + R_{1t} + R_{2t} \\ &= 0. \end{aligned}$$

Because the government spends the unrebated revenue $(1 - \rho)R_{jt}$ on the homogeneous good, this equation is the market-clearing condition for the homogeneous good. Hence the prices and quantities solved in this step form an equilibrium of the original model, and, by Step 4, household welfare differs from the objective in (17) by a constant.

A.2 Lagrangian problem and first-order conditions

This subsection derives the first-order conditions of the Ramsey problem (17). The initial stocks $n_{i,-1} > 0$ and the foreign tariff path are given; p is the constant manufacturing producer price. Maintain $\sigma > 1$, $\theta \geq 1$, $\beta, \delta \in (0, 1)$, $\chi > 0$, $b, p > 0$, and $\rho \in [0, 1]$, with positive firm stocks and interior entry along the allocation under consideration.

The consumer expenditure shares used below are

$$\begin{aligned} s_{11t} &= P_{1t}^{\sigma-1} n_{1t-1} p^{1-\sigma}, & s_{21t} &= P_{1t}^{\sigma-1} n_{2t-1} (\theta \tau_{21t} p)^{1-\sigma}, \\ s_{12t} &= P_{2t}^{\sigma-1} n_{1t-1} (\theta \tau_{12t} p)^{1-\sigma}, & s_{22t} &= P_{2t}^{\sigma-1} n_{2t-1} p^{1-\sigma}. \end{aligned}$$

Each destination's shares sum to one, $s_{11t} + s_{21t} = s_{12t} + s_{22t} = 1$.

The multipliers are current-value multipliers. The Lagrangian for (17) is

$$\begin{aligned}
\mathcal{L} = & \sum_{t=0}^{\infty} \beta^t \left[-\ln P_{1t} + n_{1t-1} \pi_{1t} - b^\chi \frac{\chi}{\chi+1} v_{1t}^{\chi+1} + \rho(1 - \tau_{21t}^{-1}) \left(1 - P_{1t}^{\sigma-1} n_{1t-1} p^{1-\sigma} \right) \right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{1t}^p \left[n_{1t-1} p^{1-\sigma} + n_{2t-1} (\theta \tau_{21t} p)^{1-\sigma} - P_{1t}^{1-\sigma} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{2t}^p \left[n_{2t-1} p^{1-\sigma} + n_{1t-1} (\theta \tau_{12t} p)^{1-\sigma} - P_{2t}^{1-\sigma} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{1t}^\pi \left[\frac{p^{1-\sigma}}{\sigma} \left(P_{1t}^{\sigma-1} + P_{2t}^{\sigma-1} \tau_{12t}^{-\sigma} \theta^{1-\sigma} \right) - \pi_{1t} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{2t}^\pi \left[\frac{p^{1-\sigma}}{\sigma} \left(P_{2t}^{\sigma-1} + P_{1t}^{\sigma-1} \tau_{21t}^{-\sigma} \theta^{1-\sigma} \right) - \pi_{2t} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{1t}^n [(1 - \delta) n_{1t-1} + (bv_{1t})^\chi - n_{1t}] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{2t}^n [(1 - \delta) n_{2t-1} + (bv_{2t})^\chi - n_{2t}] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{1t}^v [\beta(\pi_{1t+1} + (1 - \delta)v_{1t+1}) - v_{1t}] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_{2t}^v [\beta(\pi_{2t+1} + (1 - \delta)v_{2t+1}) - v_{2t}] \\
& + \sum_{t=0}^{\infty} \beta^t \lambda_t^{\tau_{LB}} (\tau_{21t} - 1).
\end{aligned}$$

Take partial derivatives with respect to prices, profits, firm values, stocks, and the home tariff as separate variables; the constraints impose their equilibrium relationships. Each first-order condition below is divided by β^t . At date zero there is no preceding firm-value constraint, so set

$$\lambda_{1,-1}^v = \lambda_{2,-1}^v = 0.$$

1. Home operating profits: $\partial \mathcal{L} / \partial \pi_{1t} = 0$. Home profits enter household income, the profit constraint, and the preceding firm-value constraint, giving

$$\lambda_{1t}^\pi = n_{1t-1} + \lambda_{1t-1}^v. \quad (\text{A.1})$$

2. Foreign operating profits: $\partial \mathcal{L} / \partial \pi_{2t} = 0$. Foreign profits enter the profit and preceding firm-value constraints, but not home household income, giving

$$\lambda_{2t}^\pi = \lambda_{2t-1}^v. \quad (\text{A.2})$$

3. Home price index: $\partial \mathcal{L} / \partial P_{1t} = 0$. The home price index enters consumption, rebated revenue, and both countries' profit constraints. Expressing the derivative in expenditure shares and

substituting (A.1)–(A.2) gives

$$\lambda_{1t}^P P_{1t}^{1-\sigma} = \frac{1}{\sigma-1} + \rho(1 - \tau_{21t}^{-1})s_{11t} - \frac{1}{\sigma} \left[\left(1 + \frac{\lambda_{1t-1}^v}{n_{1t-1}}\right) s_{11t} + \frac{\lambda_{2t-1}^v}{n_{2t-1}} \frac{s_{21t}}{\tau_{21t}} \right]. \quad (\text{A.3})$$

In (A.3), s_{21t} is divided by τ_{21t} because dividing import expenditure by τ_{21t} converts consumer expenditure into producer revenue.

4. Foreign price index: $\partial \mathcal{L} / \partial P_{2t} = 0$. The foreign price index enters the foreign price-index constraint and both profit constraints. Applying the same substitution gives

$$\lambda_{2t}^P P_{2t}^{1-\sigma} = -\frac{1}{\sigma} \left[\left(1 + \frac{\lambda_{1t-1}^v}{n_{1t-1}}\right) \frac{s_{12t}}{\tau_{12t}} + \frac{\lambda_{2t-1}^v}{n_{2t-1}} s_{22t} \right]. \quad (\text{A.4})$$

5. Home firm value: $\partial \mathcal{L} / \partial v_{1t} = 0$. A higher home firm value induces entry and raises entry expenditure. The relevant derivatives are

$$\frac{\partial(bv_{1t})^\chi}{\partial v_{1t}} = \chi b^\chi v_{1t}^{\chi-1}, \quad \frac{\partial}{\partial v_{1t}} \left[b^\chi \frac{\chi}{\chi+1} v_{1t}^{\chi+1} \right] = \chi b^\chi v_{1t}^\chi.$$

Combining these terms with the current and preceding value constraints yields

$$\lambda_{1t}^v = (1 - \delta)\lambda_{1t-1}^v + \chi b^\chi v_{1t}^{\chi-1} (\lambda_{1t}^n - v_{1t}). \quad (\text{A.5})$$

6. Foreign firm value: $\partial \mathcal{L} / \partial v_{2t} = 0$. Foreign entry costs are borne abroad. The stock and value constraints therefore give

$$\lambda_{2t}^v = (1 - \delta)\lambda_{2t-1}^v + \chi b^\chi v_{2t}^{\chi-1} \lambda_{2t}^n. \quad (\text{A.6})$$

7. Home firm stock: $\partial \mathcal{L} / \partial n_{1t} = 0$. A home firm installed at date t contributes to operating income at date $t+1$, changes both price indices and rebated revenue, and survives into subsequent periods. The first-order condition for n_{1t} is therefore

$$\lambda_{1t}^n = \beta \left[\pi_{1t+1} + \left(\lambda_{1t+1}^P - \rho(1 - \tau_{21t+1}^{-1})P_{1t+1}^{\sigma-1} \right) p^{1-\sigma} + \lambda_{2t+1}^P (\theta \tau_{12t+1} p)^{1-\sigma} + (1 - \delta)\lambda_{1t+1}^n \right]. \quad (\text{A.7})$$

8. Foreign firm stock: $\partial \mathcal{L} / \partial n_{2t} = 0$. A foreign firm installed at date t affects both price indices and survives into subsequent periods, giving

$$\lambda_{2t}^n = \beta \left[\lambda_{1t+1}^P (\theta \tau_{21t+1} p)^{1-\sigma} + \lambda_{2t+1}^P p^{1-\sigma} + (1 - \delta)\lambda_{2t+1}^n \right]. \quad (\text{A.8})$$

9. Home import tariff: $\partial \mathcal{L} / \partial \tau_{21t} = 0$. The import tariff enters rebated revenue, the home price-index constraint, the foreign profit constraint, and the tariff lower bound. Using (A.2), the first-

order condition for τ_{21t} is

$$\lambda_t^{\tau_{LB}} = \frac{(\sigma - 1)s_{21t}}{\tau_{21t}} \lambda_{1t}^P P_{1t}^{1-\sigma} + \frac{s_{21t}}{\tau_{21t}^2} \left(\frac{\lambda_{2t-1}^v}{n_{2t-1}} - \rho \right). \quad (\text{A.9})$$

Substitute (A.3) and collect the terms in expenditure shares:

$$\begin{aligned} \lambda_t^{\tau_{LB}} = \frac{s_{21t}}{\tau_{21t}} & \left[1 - \frac{\sigma - 1}{\sigma} s_{11t} + \rho(\sigma - 1)(1 - \tau_{21t}^{-1}) s_{11t} - \frac{\rho}{\tau_{21t}} \right. \\ & \left. - \frac{\sigma - 1}{\sigma} s_{11t} \frac{\lambda_{1t-1}^v}{n_{1t-1}} + \frac{1 + (\sigma - 1)s_{11t}}{\sigma \tau_{21t}} \frac{\lambda_{2t-1}^v}{n_{2t-1}} \right]. \end{aligned} \quad (\text{A.10})$$

The first line of (A.10) records the effects on current consumption, operating profits, and rebated revenue. The second line records the effects through entry induced by earlier promises.

Tariff lower bound and the interior condition. The lower-bound multiplier satisfies

$$\lambda_t^{\tau_{LB}} \geq 0, \quad \lambda_t^{\tau_{LB}} (\tau_{21t} - 1) = 0.$$

For $\tau_{21t} > 1$, the lower-bound multiplier vanishes, so the bracket in (A.10) is zero. Multiplying the bracket in (A.10) by $\sigma \tau_{21t}$ and collecting terms gives

$$\begin{aligned} \tau_{21t} & \left[\sigma + (\sigma - 1) \left(\sigma \rho - 1 - \frac{\lambda_{1t-1}^v}{n_{1t-1}} \right) s_{11t} \right] \\ & = \left(\sigma \rho - \frac{\lambda_{2t-1}^v}{n_{2t-1}} \right) [1 + (\sigma - 1)s_{11t}]. \end{aligned}$$

For a nonzero coefficient on τ_{21t} , dividing by it gives (23); otherwise the equation above is the condition. Equation (23) determines τ_{21t} only implicitly, because the shares and multipliers in it depend on the policy path.

A.3 Stationary equilibrium

This subsection derives the stationary allocation and multipliers used in the proofs of Lemma 7 and Proposition 4. Maintain positive finite stocks and firm values, strictly unsaturated entry, and the existence, convergence, and regularity assumptions stated in the main text. Time subscripts are suppressed at the stationary limit. The private allocation is first derived for general τ_{12} ; the later paragraphs set $\tau_{12} = 1$.

Private entry and profits. At the stationary limit, the equations for n_{it} and v_{it} in Step 3 of Appendix A.1 imply

$$\delta n_i = (bv_i)^\chi, \quad [1 - \beta(1 - \delta)]v_i = \beta \pi_i, \quad \frac{n_1}{n_2} = \left(\frac{\pi_1}{\pi_2} \right)^\chi.$$

Operating profits, expressed in expenditure shares, are

$$\pi_1 = \frac{s_{11} + s_{12}/\tau_{12}}{\sigma n_1}, \quad \pi_2 = \frac{s_{22} + s_{21}/\tau_{21}}{\sigma n_2}.$$

Substituting the ratio of these profits into $n_1/n_2 = (\pi_1/\pi_2)^\chi$ gives

$$\left(\frac{n_1}{n_2}\right)^{1+1/\chi} = \frac{s_{11} + s_{12}/\tau_{12}}{s_{22} + s_{21}/\tau_{21}}. \quad (\text{A.11})$$

The shares are

$$\begin{aligned} s_{11} &= \frac{n_1/n_2}{n_1/n_2 + (\theta\tau_{21})^{1-\sigma}}, & s_{21} &= \frac{(\theta\tau_{21})^{1-\sigma}}{n_1/n_2 + (\theta\tau_{21})^{1-\sigma}}, \\ s_{12} &= \frac{(n_1/n_2)(\theta\tau_{12})^{1-\sigma}}{1 + (n_1/n_2)(\theta\tau_{12})^{1-\sigma}}, & s_{22} &= \frac{1}{1 + (n_1/n_2)(\theta\tau_{12})^{1-\sigma}}. \end{aligned}$$

In π_1 and π_2 , s_{12} and s_{21} are divided by the tariffs because consumer expenditure on imports is divided by the tariff when it enters producer profits.

Stationary value multipliers. Stationarity in (A.5)–(A.6) gives

$$\delta\lambda_1^v = \chi b^\chi v_1^{\chi-1}(\lambda_1^n - v_1), \quad \delta\lambda_2^v = \chi b^\chi v_2^{\chi-1}\lambda_2^n.$$

Subtracting the stationary home value equation $[1 - \beta(1 - \delta)]v_1 = \beta\pi_1$ from the stationary version of (A.7), and writing (A.8) at the stationary limit, yields

$$\begin{aligned} [1 - \beta(1 - \delta)](\lambda_1^n - v_1) &= \beta[(\lambda_1^P - \rho(1 - \tau_{21}^{-1})P_1^{\sigma-1})p^{1-\sigma} + \lambda_2^P(\theta\tau_{12}p)^{1-\sigma}], \\ [1 - \beta(1 - \delta)]\lambda_2^n &= \beta[\lambda_1^P(\theta\tau_{21}p)^{1-\sigma} + \lambda_2^Pp^{1-\sigma}]. \end{aligned}$$

The stationary entry and value equations $\delta n_i = (bv_i)^\chi$ and $[1 - \beta(1 - \delta)]v_i = \beta\pi_i$ imply $b^\chi v_i^{\chi-1} = \delta n_i/v_i$ and $\beta/\{[1 - \beta(1 - \delta)]v_i\} = 1/\pi_i$. By the share definitions,

$$(\lambda_1^P - \rho(1 - \tau_{21}^{-1})P_1^{\sigma-1})p^{1-\sigma} + \lambda_2^P(\theta\tau_{12}p)^{1-\sigma} = \frac{1}{n_1}\{s_{11}[\lambda_1^P P_1^{1-\sigma} - \rho(1 - \tau_{21}^{-1})] + s_{12}\lambda_2^P P_2^{1-\sigma}\}.$$

Substituting these relations into $\delta\lambda_1^v = \chi b^\chi v_1^{\chi-1}(\lambda_1^n - v_1)$ and using $1/(n_1\pi_1) = \sigma/(s_{11} + s_{12}/\tau_{12})$ gives

$$\frac{\lambda_1^v}{n_1} = \chi\sigma \frac{s_{11}[\lambda_1^P P_1^{1-\sigma} - \rho(1 - \tau_{21}^{-1})] + s_{12}\lambda_2^P P_2^{1-\sigma}}{s_{11} + s_{12}/\tau_{12}}, \quad (\text{A.12})$$

$$\frac{\lambda_2^v}{n_2} = \chi\sigma \frac{s_{21}\lambda_1^P P_1^{1-\sigma} + s_{22}\lambda_2^P P_2^{1-\sigma}}{s_{22} + s_{21}/\tau_{21}}. \quad (\text{A.13})$$

The foreign equation follows from the same substitutions. Although β, δ, b, p do not appear in (A.12)–(A.13), they still affect stock levels, entry feasibility, and welfare along the transition.

Price multipliers and the optimal tariff. Set $\tau_{12} = 1$ in (A.3)–(A.4) evaluated at the stationary equilibrium. Together with (A.12)–(A.13), these equations form a linear system in $\lambda_1^P P_1^{1-\sigma}$, $\lambda_2^P P_2^{1-\sigma}$, λ_1^v/n_1 , and λ_2^v/n_2 . After eliminating λ_1^v/n_1 and λ_2^v/n_2 , the two remaining equations in $\lambda_1^P P_1^{1-\sigma}$ and $\lambda_2^P P_2^{1-\sigma}$ have determinant

$$(1 + \chi) \left[1 + \chi \frac{(s_{11} - s_{12})(s_{11}s_{22} - s_{12}s_{21}/\tau_{21})}{(s_{11} + s_{12})(s_{22} + s_{21}/\tau_{21})} \right] > 0.$$

The share formulas imply $s_{11} \geq s_{12}$ and $s_{11}s_{22} \geq s_{12}s_{21}/\tau_{21}$ for $\theta, \tau_{21} \geq 1$. Thus these multipliers are uniquely determined at each private steady state. Multiplying the stationary version of (A.9) by τ_{21}/s_{21} gives

$$\frac{\tau_{21}}{s_{21}} \lambda^{\tau_{LB}} = (\sigma - 1) \lambda_1^p P_1^{1-\sigma} + \frac{\lambda_2^v/n_2 - \rho}{\tau_{21}}. \quad (\text{A.14})$$

At an optimal stationary tariff, the right side of (A.14) is nonnegative if $\tau_{21} = 1$ and zero if $1 < \tau_{21} < \infty$.

Uniqueness and asymmetry of the private allocation. With $\tau_{12} = 1$, substituting the shares into (A.11) and cancelling the positive factor n_1/n_2 gives

$$\left(\frac{n_1}{n_2}\right)^{1/\chi} = \frac{1 + 2\theta^{1-\sigma}(n_1/n_2) + \theta^{2(1-\sigma)}\tau_{21}^{1-\sigma}}{(n_1/n_2)[1 + \theta^{2(1-\sigma)}\tau_{21}^{-\sigma}] + \theta^{1-\sigma}\tau_{21}^{1-\sigma}(1 + \tau_{21}^{-1})}. \quad (\text{A.15})$$

The left side increases strictly with n_1/n_2 . The numerator of the derivative of the right side is

$$-[1 - \theta^{2(1-\sigma)}\tau_{21}^{1-\sigma}][1 - \theta^{2(1-\sigma)}\tau_{21}^{-\sigma}] \leq 0.$$

The right side of (A.15) is positive at $n_1/n_2 = 0$, does not increase with n_1/n_2 , and is bounded, so (A.15) has exactly one positive solution n_1/n_2 . At $n_1/n_2 = 1$, the numerator of the right side of (A.15) minus its denominator equals

$$\theta^{1-\sigma}[2 - \tau_{21}^{1-\sigma} - \tau_{21}^{-\sigma}] + \theta^{2(1-\sigma)}\tau_{21}^{-\sigma}(\tau_{21} - 1).$$

This is positive when $\tau_{21} > 1$ and zero at free trade. Hence

$$\tau_{21} > 1 \implies \frac{n_1}{n_2} > 1, \quad \tau_{21} = 1 \implies \frac{n_1}{n_2} = 1. \quad (\text{A.16})$$

A.3.1 Entry costs and the stationary tariff in stock values

In this section, we establish the stationary tariff formula in (26)–(27). Wages are one and entry is strictly unsaturated, $0 < bv_{it} < 1$.

The entry cost of the marginal home entrant. A potential entrant with cost ξ enters if $\xi \leq v_{1t}$. The entry cost ξ has density $\chi b^\chi \xi^{\chi-1}$ on $[0, 1/b]$. The mass of entrants and their total entry cost are

$$\int_0^{v_{1t}} \chi b^\chi \xi^{\chi-1} d\xi = (bv_{1t})^\chi, \quad \int_0^{v_{1t}} \xi \chi b^\chi \xi^{\chi-1} d\xi = b^\chi \frac{\chi}{1+\chi} v_{1t}^{1+\chi}.$$

Their derivatives with respect to v_{1t} imply

$$\frac{\frac{d}{dv_{1t}} \left(b^\chi \frac{\chi}{1+\chi} v_{1t}^{1+\chi} \right)}{\frac{d}{dv_{1t}} (bv_{1t})^\chi} = \frac{\chi b^\chi v_{1t}^\chi}{\chi b^\chi v_{1t}^{\chi-1}} = v_{1t}. \quad (\text{A.17})$$

The current-value stock multiplier λ_{1t}^n values another home firm at the end of date t . Thus

$$\frac{\partial}{\partial v_{1t}} \left[-\frac{\chi b^\chi}{1+\chi} v_{1t}^{1+\chi} + \lambda_{1t}^n (bv_{1t})^\chi \right] = \chi b^\chi v_{1t}^{\chi-1} (\lambda_{1t}^n - v_{1t}).$$

By (A.17), the term v_{1t} subtracted from λ_{1t}^n is the entry cost of the marginal home entrant. Foreign entry costs are borne abroad, so the contribution of foreign entry to home welfare is $\chi b^\chi v_{2t}^{\chi-1} \lambda_{2t}^n$.

Stationary normalization and tariff choice. At the stationary limit, (A.5)–(A.6) give

$$\delta \lambda_1^v = \chi b^\chi v_1^{\chi-1} (\lambda_1^n - v_1), \quad \delta \lambda_2^v = \chi b^\chi v_2^{\chi-1} \lambda_2^n.$$

Using $(bv_i)^\chi = \delta n_i$ and dividing by δn_i gives (26). At an interior stationary tariff, where the bracket in (A.10) is zero, substitute these identities into (A.10) and multiply the bracket by $\sigma \tau_{21}$:

$$\tau_{21} \left\{ \sigma + (\sigma - 1) \left[\sigma \rho - 1 - \chi \left(\frac{\lambda_1^n}{v_1} - 1 \right) \right] s_{11} \right\} = \left(\sigma \rho - \chi \frac{\lambda_2^n}{v_2} \right) [1 + (\sigma - 1) s_{11}]. \quad (\text{A.18})$$

Dividing (A.18) by $\sigma + (\sigma - 1) \left[\sigma \rho - 1 - \chi \left(\frac{\lambda_1^n}{v_1} - 1 \right) \right] s_{11}$, when it is nonzero, gives (27).

A.4 Entry along the commitment path

In this section, we derive the expressions for λ_{1t-1}^v and λ_{2t-1}^v in (24)–(25) and the direction of reoptimization in (33). Take the foreign tariff path as given. Maintain positive stocks and values and strictly unsaturated entry. The value multipliers satisfy (A.5)–(A.6), with $\lambda_{i,-1}^v = 0$.

Earlier entry and promised profits. Iterating (A.5)–(A.6) from $\lambda_{i,-1}^v = 0$ gives

$$\begin{aligned} \lambda_{1t-1}^v &= \sum_{k=0}^{t-1} (1-\delta)^{t-1-k} \chi b^\chi v_{1k}^{\chi-1} (\lambda_{1k}^n - v_{1k}), \\ \lambda_{2t-1}^v &= \sum_{k=0}^{t-1} (1-\delta)^{t-1-k} \chi b^\chi v_{2k}^{\chi-1} \lambda_{2k}^n. \end{aligned} \quad (\text{A.19})$$

In (A.19), the weight on date- k entry combines firm survival, $(1-\delta)^{t-1-k}$, with the derivative of entry with respect to firm value, $\chi b^\chi v_{ik}^{\chi-1}$. The net social value of home entry is $\lambda_{1k}^n - v_{1k}$; the value of foreign entry to home households is λ_{2k}^n because its cost is borne abroad. Appendix A.3.1 shows that v_{1k} in $\lambda_{1k}^n - v_{1k}$ is the entry cost of the marginal home entrant.

At fixed inherited stocks, the share derivatives are

$$\frac{\partial s_{11t}}{\partial \tau_{21t}} = \frac{(\sigma - 1) s_{11t} s_{21t}}{\tau_{21t}}, \quad \frac{\partial s_{21t}}{\partial \tau_{21t}} = -\frac{(\sigma - 1) s_{11t} s_{21t}}{\tau_{21t}}.$$

The foreign-market shares s_{12t} and s_{22t} do not depend on the current home tariff. Differentiating

$\sigma n_{1t-1} \pi_{1t} = s_{11t} + s_{12t} / \tau_{12t}$ and $\sigma n_{2t-1} \pi_{2t} = s_{22t} + s_{21t} / \tau_{21t}$ yields

$$\begin{aligned} \frac{\partial \pi_{1t}}{\partial \tau_{21t}} &= \frac{(\sigma - 1) s_{11t} s_{21t}}{\sigma \tau_{21t} n_{1t-1}} > 0, \\ \frac{\partial \pi_{2t}}{\partial \tau_{21t}} &= \frac{1}{\sigma n_{2t-1}} \left(\frac{1}{\tau_{21t}} \frac{\partial s_{21t}}{\partial \tau_{21t}} - \frac{s_{21t}}{\tau_{21t}^2} \right) \\ &= -\frac{s_{21t} [1 + (\sigma - 1) s_{11t}]}{\sigma \tau_{21t}^2 n_{2t-1}} < 0. \end{aligned} \quad (\text{A.20})$$

Substituting (A.19) and (A.20) into $\sum_{i=1}^2 \lambda_{it-1}^v \partial \pi_{it} / \partial \tau_{21t}$ gives

$$\begin{aligned} \sum_{i=1}^2 \lambda_{it-1}^v \frac{\partial \pi_{it}}{\partial \tau_{21t}} &= \frac{s_{21t}}{\sigma \tau_{21t}^2} \left\{ \frac{(\sigma - 1) \tau_{21t} s_{11t}}{n_{1t-1}} \sum_{k=0}^{t-1} (1 - \delta)^{t-1-k} \chi b^\chi v_{1k}^{\chi-1} (\lambda_{1k}^n - v_{1k}) \right. \\ &\quad \left. - \frac{1 + (\sigma - 1) s_{11t}}{n_{2t-1}} \sum_{k=0}^{t-1} (1 - \delta)^{t-1-k} \chi b^\chi v_{2k}^{\chi-1} \lambda_{2k}^n \right\}. \end{aligned} \quad (\text{A.21})$$

Beneficial home entry ($\lambda_{1k}^n - v_{1k} > 0$) supports promised protection; beneficial foreign entry ($\lambda_{2k}^n > 0$) raises the cost of protection. When both are beneficial, the two weighted sums in (A.21) determine the sign of the net effect.

Reoptimization at inherited stocks. At a positive interior announced tariff, the first-order condition (A.9) with $\lambda_t^{\tau_{LB}} = 0$ can be written as

$$\frac{\partial}{\partial \tau_{21t}} \left[-\ln P_{1t} + n_{1t-1} \pi_{1t} + \rho(1 - \tau_{21t}^{-1}) s_{21t} \right] = -\sum_{i=1}^2 \lambda_{it-1}^v \frac{\partial \pi_{it}}{\partial \tau_{21t}}. \quad (\text{A.22})$$

The sum $\sum_{i=1}^2 \lambda_{it-1}^v \partial \pi_{it} / \partial \tau_{21t}$ has the same sign as the left side minus the right side of (33). If this sum is positive, the left side of (A.22) is negative at the announced tariff, so reoptimizing at date t lowers the current tariff. If this sum is negative, the left side of (A.22) is positive, so reoptimizing raises the current tariff. This argument uses the fact that $-\ln P_{1t} + n_{1t-1} \pi_{1t} + \rho(1 - \tau_{21t}^{-1}) s_{21t}$ has a unique maximizer in τ_{21t} , which we establish in the proof of Proposition 3 in Appendix B below.

To check that this reoptimization is feasible, change only the date- t tariff and retain the continuation from $t + 1$ onward. Firm value $v_{it} = \beta[\pi_{it+1} + (1 - \delta)v_{it+1}]$ is unchanged, as are current entry, its cost, and n_{it} . Current prices, profits, and revenue adjust, while all future equilibrium conditions remain satisfied. The value equations for dates before t belong to the earlier promises and do not constrain a government that reoptimizes at date t .

For any feasible continuation, the part of welfare that depends on τ_{21t} is

$$-\ln P_{1t} + \frac{s_{11t}}{\sigma} + \rho(1 - \tau_{21t}^{-1}) s_{21t}.$$

Home export profits are included in total welfare but are constant in this choice. The maximizer of this expression is therefore the first tariff of the newly optimized plan even if future promises are also revised. If the sum in (A.22) is nonzero at a regular stationary limit, then by continuity it is nonzero at all sufficiently late dates, so reoptimizing at those dates changes the tariff and raises welfare; this establishes time inconsistency.⁸

⁸This comparison concerns commitment and reoptimization at the same inherited stocks; it does not establish mono-

B Proofs

B.1 Proof of Lemma 1

These claims follow from direct calculation. For the first item:

$$\begin{aligned}
& n_{1t-1} (p_{11t}^c q_{11t} + p_{12t}^c q_{12t}) + n_{2t-1} (p_{22t}^c q_{22t} + p_{21t}^c q_{21t}) \\
&= n_{1t-1} (p_{11t}^x q_{11t} + \tau_{12t} p_{12t}^x q_{12t}) + n_{2t-1} (p_{22t}^x q_{22t} + \tau_{21t} p_{21t}^x q_{21t}) \\
&= n_{1t-1} \left(p C_{1t}^m \left(\frac{p}{P_{1t}} \right)^{-\sigma} + \tau_{12t} p \theta C_{2t}^m \left(\frac{\tau_{12t} p \theta}{P_{2t}} \right)^{-\sigma} \right) + n_{2t-1} \left(p C_{2t}^m \left(\frac{p}{P_{2t}} \right)^{-\sigma} + \tau_{21t} p \theta C_{1t}^m \left(\frac{\tau_{21t} p \theta}{P_{1t}} \right)^{-\sigma} \right) \\
&= n_{1t-1} \left(\frac{p^{1-\sigma}}{P_{1t}^{1-\sigma}} + \frac{(\tau_{12t} p \theta)^{1-\sigma}}{P_{2t}^{1-\sigma}} \right) + n_{2t-1} \left(\frac{p^{1-\sigma}}{P_{2t}^{1-\sigma}} + \frac{(\tau_{21t} p \theta)^{1-\sigma}}{P_{1t}^{1-\sigma}} \right) \\
&= n_{1t-1} \frac{p^{1-\sigma}}{P_{1t}^{1-\sigma}} + n_{2t-1} \frac{(\tau_{21t} p \theta)^{1-\sigma}}{P_{1t}^{1-\sigma}} + n_{1t-1} \frac{(\tau_{12t} p \theta)^{1-\sigma}}{P_{2t}^{1-\sigma}} + n_{2t-1} \frac{p^{1-\sigma}}{P_{2t}^{1-\sigma}} = 2
\end{aligned}$$

where the first equality uses $p_{ijt}^c = \tau_{ijt} p_{ijt}^x$, the second equality uses the CES demand functions (5) and the optimal producer price (8), the third equality uses that total manufacturing expenditure in each country is 1 (equation (2)), the fourth equality rearranges, and the last equality uses the definitions and properties of expenditure shares (equations 16).

For the second item, the proof is similar. It additionally requires the definition of government revenues (equation (15)).

For the third item, show first that profits (equation (9)) are a constant fraction of manufacturing sales

$$\pi_{it} = \frac{1}{\sigma} \left[p_{iit}^x q_{iit} + p_{ijt}^x q_{ijt} \right] \quad \text{for } j \neq i.$$

The remainder of the proof then directly follows from item (ii).

B.2 Proof of Lemma 2

In steady state the accumulation equation (12) and the value of a firm (10) become

$$\delta n_j = (b v_j)^\chi, \quad v_j = \frac{\beta}{1 - \beta(1 - \delta)} \pi_j$$

Combining these gives

$$n_j^{\frac{1}{\chi}} = \frac{1}{\delta^{\frac{1}{\chi}}} \frac{b\beta}{1 - \beta(1 - \delta)} \pi_j. \quad (\text{B.1})$$

tonicity along the original path.

Then

$$\begin{aligned}
n_1^{\frac{\chi+1}{\chi}} + n_2^{\frac{\chi+1}{\chi}} &= n_1 n_1^{\frac{1}{\chi}} + n_2 n_2^{\frac{1}{\chi}} \\
&= n_1 \frac{1}{\delta^{\frac{1}{\chi}}} \frac{b\beta}{1 - \beta(1 - \delta)} \pi_1 + n_2 \frac{1}{\delta^{\frac{1}{\chi}}} \frac{b\beta}{1 - \beta(1 - \delta)} \pi_2 \\
&= \frac{1}{\delta^{\frac{1}{\chi}}} \frac{b\beta}{1 - \beta(1 - \delta)} (n_1 \pi_1 + n_2 \pi_2) \\
&= \frac{1}{\delta^{\frac{1}{\chi}}} \frac{b\beta}{1 - \beta(1 - \delta)} \frac{1}{\sigma} [2 - R_1 - R_2]
\end{aligned}$$

where the last equality uses Lemma 1.

B.3 Proof of Lemma 3

In steady state, total profits in the economy are

$$\begin{aligned}
\Pi_1 + \Pi_2 &= n_1 \pi_1 - b^\chi \frac{\chi}{\chi + 1} (v_1)^{\chi+1} + n_2 \pi_2 - b^\chi \frac{\chi}{\chi + 1} (v_2)^{\chi+1} \\
&= \left(\frac{1 - \beta(1 - \delta)}{\beta} - \frac{\chi\delta}{\chi + 1} \right) \frac{\delta^{\frac{1}{\chi}}}{b} (n_1)^{\frac{\chi+1}{\chi}} + \left(\frac{1 - \beta(1 - \delta)}{\beta} - \frac{\chi\delta}{\chi + 1} \right) \frac{\delta^{\frac{1}{\chi}}}{b} (n_2)^{\frac{\chi+1}{\chi}} \\
&= \left(\frac{1 - \beta(1 - \delta)}{\beta} - \frac{\chi\delta}{\chi + 1} \right) \frac{\delta^{\frac{1}{\chi}}}{b} \left((n_1)^{\frac{\chi+1}{\chi}} + (n_2)^{\frac{\chi+1}{\chi}} \right),
\end{aligned}$$

where the first equality uses the definition of aggregate profits (14) for both countries, and the second uses the accumulation equation and the value of a firm in steady state (equation (B.1)). The claim now follows from Lemma 2.

B.4 Proof of Corollary 1

Total household income in steady state, $Y_1 + Y_2$, is

$$\begin{aligned}
&L_1 + L_2 + \Pi_1 + \Pi_2 + \rho R_1 + \rho R_2 \\
&= L_1 + L_2 + \frac{1}{\sigma} \left(1 - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)} \right) [2 - R_1 - R_2] + \rho R_1 + \rho R_2 \\
&= L_1 + L_2 + \frac{2}{\sigma} \left(1 - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)} \right) - \frac{1}{\sigma} \left(1 - \sigma\rho - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)} \right) (R_1 + R_2)
\end{aligned}$$

where the definition of household income follows from budget constraint (1). The first equality uses Lemma 3, and the second rearranges. For the last claim, note that

$$\frac{\partial (Y_1 + Y_2)}{\partial (R_1 + R_2)} = -\frac{1}{\sigma} \left(1 - \sigma\rho - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)} \right),$$

which is nonnegative if and only if

$$\rho \geq \frac{1}{\sigma} \left(1 - \frac{\chi}{\chi + 1} \frac{\delta\beta}{1 - \beta(1 - \delta)} \right).$$

B.5 Proof of Proposition 2

By Proposition 1, τ_{21t} maximizes the date- t objective whose derivative is given in (19), taking (n_{1t-1}, n_{2t-1}) as given. Collecting terms, (19) equals

$$\frac{s_{21t}}{\tau_{21t}^2} \left\{ \rho [1 + (\sigma - 1)s_{11t}] - \tau_{21t} \left[1 - \frac{\sigma - 1}{\sigma} s_{11t} + \rho(\sigma - 1)s_{11t} \right] \right\}.$$

The coefficient of τ_{21t} in braces exceeds $1/\sigma$ because $s_{11t} < 1$. Since $s_{21t}/\tau_{21t}^2 > 0$, (19) has the sign of the term in braces, which is zero exactly when (20) holds.

Using $\partial s_{11t}/\partial \tau_{21t} = (\sigma - 1)s_{11t}s_{21t}/\tau_{21t}$, at any zero the derivative of the term in braces with respect to τ_{21t} equals

$$-\frac{((\sigma - 1)s_{11t})^2 - (\sigma(1 - \rho) - 1)(\sigma - 1)s_{11t} + 1}{1 + (\sigma - 1)s_{11t}}.$$

At a zero with $\tau_{21t} \geq 1$, (20) implies $(\sigma - 1)s_{11t} \geq \sigma(1 - \rho)$, so the numerator is at least $(\sigma - 1)s_{11t} + 1 > 0$. Hence the term in braces crosses zero at most once on $\tau_{21t} \geq 1$, and from above. As $\tau_{21t} \rightarrow \infty$, the term in braces tends to $-\infty$, because $\rho[1 + (\sigma - 1)s_{11t}]$ is bounded and the coefficient of τ_{21t} exceeds $1/\sigma$.

At $\tau_{21t} = 1$ the term in braces equals $\rho - 1 + \frac{\sigma - 1}{\sigma} \frac{n_{1t-1}}{n_{2t-1}\theta^{1-\sigma} + n_{1t-1}}$. If this is positive, the objective increases up to the unique solution of (20) and decreases afterwards. Otherwise the term in braces is negative for all $\tau_{21t} > 1$, because the term in braces can cross zero only from above; the objective then decreases in τ_{21t} , and $\tau_{21t} = 1$.

B.6 Proof of Proposition 3

Step 1: take the derivative with respect to the tariff. Rearranging (29) gives

$$\begin{aligned} & \frac{\partial}{\partial \tau_{21t}} \left[-\ln P_{1t} + \frac{s_{11t}}{\sigma} + \rho(1 - \tau_{21t}^{-1})s_{21t} \right] \\ &= \frac{s_{21t}}{\tau_{21t}^2} \left\{ \rho[1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}] - \tau_{21t} \left[1 - \frac{\sigma - 1}{\sigma} s_{11t} \right] \right\}. \end{aligned} \quad (\text{B.2})$$

Because $\tau_{21t}[1 - (\sigma - 1)s_{11t}/\sigma] > 0$, the derivative in (B.2) is negative whenever $1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t} \leq 0$, so an interior solution has $1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t} > 0$. Setting the derivative in (B.2) to zero gives

$$\rho = \frac{\tau_{21t}[1 - (\sigma - 1)s_{11t}/\sigma]}{1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}}.$$

Step 2: show that the rebate required for an interior solution increases with the tariff. The denominator $1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}$ starts at one and decreases strictly in τ_{21t} , since

$$\begin{aligned} \frac{\partial}{\partial \tau_{21t}} [1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}] &= -(\sigma - 1)s_{11t} \\ &\quad - \frac{(\sigma - 1)^2(\tau_{21t} - 1)}{\tau_{21t}} s_{11t}s_{21t} < 0. \end{aligned}$$

It eventually becomes negative because as $\tau_{21t} \rightarrow \infty$, $s_{11t} \rightarrow 1$ and $s_{21t} \rightarrow 0$. On the interval of τ_{21t}

where it remains positive, the quotient rule and the share derivatives in Appendix A.4 give

$$\begin{aligned}
& \frac{\partial}{\partial \tau_{21t}} \left\{ \frac{\tau_{21t}[1 - (\sigma - 1)s_{11t}/\sigma]}{1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}} \right\} \\
&= \frac{\left[1 - \frac{\sigma - 1}{\sigma}s_{11t} - \frac{(\sigma - 1)^2}{\sigma}s_{11t}s_{21t} \right] [1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}]}{[1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}]^2} \\
&\quad + \frac{\tau_{21t}[1 - (\sigma - 1)s_{11t}/\sigma] \left[(\sigma - 1)s_{11t} + \frac{(\sigma - 1)^2(\tau_{21t} - 1)}{\tau_{21t}}s_{11t}s_{21t} \right]}{[1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}]^2} \\
&= \frac{1 + (\sigma - 1)^2(\tau_{21t} - 1)s_{11t}s_{21t}}{[1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}]^2} > 0.
\end{aligned} \tag{B.3}$$

Thus, for fixed ρ , the derivative in (B.2) can cross zero only once, from positive to negative. The right side of the condition for ρ in Step 1 equals $1 - (\sigma - 1)s_{11t}/\sigma$ at free trade and goes to positive infinity when its denominator approaches zero from above. At tariffs where this denominator is zero or negative, the derivative in (B.2) is already strictly negative by Step 1. This gives the unique solution: free trade when (31) holds, and a finite interior tariff when that inequality is reversed. At equality, every strictly larger tariff lowers current welfare, so free trade remains the unique optimum.

Step 3: show that the second derivative is strictly negative at an interior solution. Where the denominator is positive, factor (B.2) as

$$\frac{s_{21t}}{\tau_{21t}^2} [1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}] \times \left\{ \rho - \frac{\tau_{21t}[1 - (\sigma - 1)s_{11t}/\sigma]}{1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}} \right\}.$$

The expression in braces vanishes at the optimal tariff. Hence, by (B.3),

$$\frac{\partial^2}{\partial \tau_{21t}^2} \left[-\ln P_{1t} + \frac{s_{11t}}{\sigma} + \rho(1 - \tau_{21t}^{-1})s_{21t} \right] = -\frac{s_{21t}}{\tau_{21t}^2} \frac{1 + (\sigma - 1)^2(\tau_{21t} - 1)s_{11t}s_{21t}}{1 - (\sigma - 1)(\tau_{21t} - 1)s_{11t}} < 0. \tag{B.4}$$

The strict inequality follows analytically for every interior first-period or reset optimum.

B.7 Proof of Lemma 6

There are no inherited value multipliers at date zero. At any given positive stocks, the first tariff solves (28); the proof of Proposition 3 shows that this solution is unique. The derivative of the objective in (28) at free trade is nonpositive if and only if

$$\rho \leq 1 - \frac{\sigma - 1}{\sigma} \frac{n_{1,-1}}{n_{1,-1} + n_{2,-1}\theta^{1-\sigma}}.$$

This is the necessary and sufficient condition for $\tau_{21,0} = 1$ in Proposition 3, including equality. At $n_{1,-1} = n_{2,-1}$ it reduces to (34) in Lemma 6.

Under the strict reverse inequality, the unique optimum is interior and $\rho > 1/\sigma$. In (30), the derivative of the right side with respect to $s_{11,0}$ is

$$\frac{\sigma\rho(\sigma - 1)[1 + \sigma(1 - \rho)]}{[\sigma + (\sigma - 1)(\sigma\rho - 1)s_{11,0}]^2} > 0.$$

Since $0 < s_{11,0} < 1$, evaluation at $s_{11,0} = 1$ gives

$$1 < \tau_{21,0} < \frac{\sigma^2 \rho}{1 + \sigma(\sigma - 1)\rho} \leq \frac{\sigma^2}{\sigma^2 - \sigma + 1}.$$

This proves (35). Conditional on inherited stocks, entry elasticity does not enter the current choice.

B.8 Proof of Lemma 7

The stationary equilibrium conditions and multiplier equations used below are derived in Appendix A.3.

B.8.1 Part (i): Zero long-run protection

Fix σ, χ, ρ with $\rho < 1/[\sigma(1 + \chi)]$, as in part (i) of Lemma 7. Choose a finite θ satisfying $\theta^{1-\sigma} < 1/16$ and

$$\theta^{1-\sigma} \left[1 + (\sigma - 1)\rho + \frac{8\chi}{1 + \chi} \left(5 + \frac{6}{\sigma - 1} + \sigma\rho \right) \right] < \frac{1}{\sigma(1 + \chi)} - \rho. \quad (\text{B.5})$$

The positive right side is independent of θ , while the left side tends to zero as trade costs increase. Thus every θ that satisfies $\theta^{1-\sigma} < 1/16$ and (B.5) is large enough for part (i) of the lemma.

As $\theta^{1-\sigma} \rightarrow 0$, the right side of (A.14) tends to $(\sigma - 1)\rho + 1/[\sigma(1 + \chi)] - \sigma\rho/\tau_{21}$; see Appendix B.8.2. For every $\tau_{21} \geq 1$, this limit satisfies

$$(\sigma - 1)\rho + \frac{1}{\sigma(1 + \chi)} - \frac{\sigma\rho}{\tau_{21}} \geq \frac{1}{\sigma(1 + \chi)} - \rho.$$

Applying (B.10) and the strict bound above makes the right side of (A.14) strictly positive at every finite tariff. Hence no tariff above one satisfies (A.14) with $\lambda^{\tau_{LB}} = 0$, and at free trade (A.14) gives a positive lower-bound multiplier, as the first-order conditions for free trade require. Appendix B.8.4 rules out tariffs diverging to infinity. Thus every regular convergent optimal allocation has $\tau_{21} = 1$.

B.8.2 Uniform bounds for large trade costs

The following bounds are used in Appendix B.8.1 and Appendix B.9.2. Maintain $\tau_{12} = 1$, $0 \leq \rho \leq 1$, finite $\chi > 0$, and $0 < \theta^{1-\sigma} < 1/16$. The bounds below hold for every finite $\tau_{21} \geq 1$.

By (A.16), $n_1/n_2 \geq 1$. Bounding the denominator of the right side of (A.15) below by n_1/n_2 gives

$$1 \leq (n_1/n_2)^{1/\chi} \leq \frac{1 + \theta^{2(1-\sigma)}}{n_1/n_2} + 2\theta^{1-\sigma}.$$

Rearrangement yields

$$1 \leq \frac{n_1}{n_2} \leq \frac{1 + \theta^{2(1-\sigma)}}{1 - 2\theta^{1-\sigma}} < 2. \quad (\text{B.6})$$

Thus $s_{21} \leq \theta^{1-\sigma}$, $s_{12} \leq 2\theta^{1-\sigma}$, $s_{11} \geq 1 - \theta^{1-\sigma}$, and $s_{22} \geq 1 - 2\theta^{1-\sigma}$.

Substitute the stationary versions of (A.3)–(A.4) into (A.12)–(A.13) and collect the terms in

λ_1^v/n_1 and λ_2^v/n_2 :

$$\begin{aligned} & \left[1 + \chi \frac{s_{11}^2 + s_{12}^2}{s_{11} + s_{12}} \right] \frac{\lambda_1^v}{n_1} + \chi \frac{s_{11}s_{21}/\tau_{21} + s_{12}s_{22}}{s_{11} + s_{12}} \frac{\lambda_2^v}{n_2} \\ &= \chi \left[\frac{\sigma s_{11}}{(\sigma - 1)(s_{11} + s_{12})} - \frac{\sigma \rho (1 - \tau_{21}^{-1}) s_{11}s_{21}}{s_{11} + s_{12}} - \frac{s_{11}^2 + s_{12}^2}{s_{11} + s_{12}} \right]. \end{aligned} \quad (\text{B.7})$$

$$\begin{aligned} & \chi \frac{s_{21}s_{11} + s_{22}s_{12}}{s_{22} + s_{21}/\tau_{21}} \frac{\lambda_1^v}{n_1} + \left[1 + \chi \frac{s_{21}^2/\tau_{21} + s_{22}^2}{s_{22} + s_{21}/\tau_{21}} \right] \frac{\lambda_2^v}{n_2} \\ &= \chi \left[\frac{\sigma s_{21}}{(\sigma - 1)(s_{22} + s_{21}/\tau_{21})} + \frac{\sigma \rho (1 - \tau_{21}^{-1}) s_{21}s_{11}}{s_{22} + s_{21}/\tau_{21}} - \frac{s_{21}s_{11} + s_{22}s_{12}}{s_{22} + s_{21}/\tau_{21}} \right]. \end{aligned} \quad (\text{B.8})$$

These equations hold at every finite $\theta \geq 1$; as $\theta^{1-\sigma} \rightarrow 0$, they give

$$(1 + \chi)\lambda_1^v/n_1 = \chi/(\sigma - 1), \quad (1 + \chi)\lambda_2^v/n_2 = 0.$$

Equations (B.7)–(B.8) are linear in λ_1^v/n_1 and λ_2^v/n_2 . In (B.7), the coefficients of λ_1^v/n_1 and λ_2^v/n_2 are $1 + \chi(s_{11}^2 + s_{12}^2)/(s_{11} + s_{12})$ and $\chi(s_{11}s_{21}/\tau_{21} + s_{12}s_{22})/(s_{11} + s_{12})$; in (B.8), they are $\chi(s_{21}s_{11} + s_{22}s_{12})/(s_{22} + s_{21}/\tau_{21})$ and $1 + \chi(s_{21}^2/\tau_{21} + s_{22}^2)/(s_{22} + s_{21}/\tau_{21})$. As $\theta^{1-\sigma} \rightarrow 0$, s_{11} and s_{22} tend to one and s_{12} and s_{21} tend to zero, so the coefficient of λ_1^v/n_1 in (B.7) and that of λ_2^v/n_2 in (B.8) tend to $1 + \chi$, and the other two coefficients tend to zero. The share bounds imply that each of the four coefficients is within $4\chi\theta^{1-\sigma}$ of its limit. For example, the coefficient of λ_1^v/n_1 in (B.7) falls short of $1 + \chi$ by χ times

$$1 - \frac{s_{11}^2 + s_{12}^2}{s_{11} + s_{12}} = \frac{s_{11}s_{21} + s_{12}s_{22}}{s_{11} + s_{12}} \leq \frac{3\theta^{1-\sigma}}{1 - \theta^{1-\sigma}} < 4\theta^{1-\sigma},$$

and the other three coefficients are bounded in the same way. In each equation, the two coefficients therefore differ from their limits by at most $8\chi\theta^{1-\sigma}$ in total. Subtracting $\chi/(\sigma - 1)$ from the right side of (B.7) leaves a term of absolute value at most $\chi\theta^{1-\sigma}[4\sigma/(\sigma - 1) + 6 + 2\sigma\rho]$, because $s_{11} + s_{12} > 1/2$; the right side of (B.8) is smaller in absolute value. Now substitute the limits $\chi/[(\sigma - 1)(1 + \chi)]$ and 0 for λ_1^v/n_1 and λ_2^v/n_2 in (B.7)–(B.8). Because the coefficients differ from their limits, the left side of each equation then differs from its limiting value by at most $8\chi\theta^{1-\sigma}/(\sigma - 1)$. Hence the two sides of each equation differ by at most $2\chi\theta^{1-\sigma}[5 + 6/(\sigma - 1) + \sigma\rho]$. Subtracting the equations at the limits from (B.7)–(B.8) and using the coefficient bounds gives

$$(1 + \chi - 8\chi\theta^{1-\sigma}) \max \left\{ \left| \frac{\lambda_1^v}{n_1} - \frac{\chi}{(\sigma - 1)(1 + \chi)} \right|, \left| \frac{\lambda_2^v}{n_2} \right| \right\} \leq 2\chi\theta^{1-\sigma} \left[5 + \frac{6}{\sigma - 1} + \sigma\rho \right].$$

Since $\theta^{1-\sigma} < 1/16$, $1 + \chi - 8\chi\theta^{1-\sigma} > (1 + \chi)/2$, which yields

$$\max \left\{ \left| \frac{\lambda_1^v}{n_1} - \frac{\chi}{(\sigma - 1)(1 + \chi)} \right|, \left| \frac{\lambda_2^v}{n_2} \right| \right\} \leq \frac{4\chi\theta^{1-\sigma}}{1 + \chi} \left[5 + \frac{6}{\sigma - 1} + \sigma\rho \right]. \quad (\text{B.9})$$

In the right side of (A.14), substitute (A.3) for $\lambda_1^p P_1^{1-\sigma}$ and replace $s_{11}, s_{21}, \lambda_1^v/n_1$ and λ_2^v/n_2 by their limits 1, 0, $\chi/[(\sigma - 1)(1 + \chi)]$ and 0. The result is $(\sigma - 1)\rho + 1/[\sigma(1 + \chi)] - \sigma\rho/\tau_{21}$. Using the actual values of s_{11} and s_{21} instead of their limits changes this result by at most $\theta^{1-\sigma}[1 + (\sigma - 1)\rho]$;

using the actual values of λ_1^v/n_1 and λ_2^v/n_2 changes it by at most twice the bound in (B.9). Hence

$$\begin{aligned} & \left| (\sigma - 1)\lambda_1^P P_1^{1-\sigma} + \frac{\lambda_2^v/n_2 - \rho}{\tau_{21}} - \left[(\sigma - 1)\rho + \frac{1}{\sigma(1 + \chi)} - \frac{\sigma\rho}{\tau_{21}} \right] \right| \\ & \leq \theta^{1-\sigma} \left[1 + (\sigma - 1)\rho + \frac{8\chi}{1 + \chi} \left(5 + \frac{6}{\sigma - 1} + \sigma\rho \right) \right]. \end{aligned} \quad (\text{B.10})$$

The bound (B.10) holds at every finite $\tau_{21} \geq 1$, with the stationary stocks and multipliers evaluated at that tariff.

B.8.3 Part (ii): Positive long-run protection

This subsection proves part (ii) of Lemma 7. If stationary free trade were the limit, the unique private allocation would have $n_1 = n_2$ and

$$s_{11} = s_{22} = \frac{1}{1 + \theta^{1-\sigma}}, \quad s_{12} = s_{21} = \frac{\theta^{1-\sigma}}{1 + \theta^{1-\sigma}}.$$

These shares are used only to examine the optimality of free trade. A positive stationary tariff generally has asymmetric stocks. At these shares, the stationary versions of (A.3), (A.4), (A.12) and (A.13) give

$$\begin{aligned} \lambda_1^P P_1^{1-\sigma} + \lambda_2^P P_2^{1-\sigma} &= \frac{1}{\sigma(\sigma - 1)(1 + \chi)}, \\ \lambda_1^P P_1^{1-\sigma} - \lambda_2^P P_2^{1-\sigma} &= \frac{1/(\sigma - 1) - (s_{11} - s_{21})/\sigma}{1 + \chi(s_{11} - s_{21})^2}. \end{aligned}$$

Substituting $\lambda_1^P P_1^{1-\sigma}$ and $\lambda_2^P P_2^{1-\sigma}$ from these two equations into (A.13) and the tariff condition (A.14) gives

$$\lambda^{\tau_{LB}} = s_{21} \left[\frac{\sigma - 1 + \chi\sigma}{2\sigma(\sigma - 1)(1 + \chi)} + \frac{[\sigma - 1 - \chi\sigma(s_{11} - s_{21})][\sigma - (\sigma - 1)(s_{11} - s_{21})]}{2\sigma(\sigma - 1)[1 + \chi(s_{11} - s_{21})^2]} - \rho \right]. \quad (\text{B.11})$$

Free trade requires $\lambda^{\tau_{LB}} \geq 0$. Simplifying the two fractions inside the brackets of (B.11) gives

$$\frac{\lambda^{\tau_{LB}}}{s_{21}} = \frac{\sigma}{2(\sigma - 1)} - \rho - \frac{1}{2\sigma(\sigma - 1)(1 + \chi)} - \frac{(s_{11} - s_{21})[(\sigma - 1)^2 + \sigma^2\chi]}{2\sigma(\sigma - 1)[1 + \chi(s_{11} - s_{21})^2]}, \quad s_{11} - s_{21} = \frac{1 - \theta^{1-\sigma}}{1 + \theta^{1-\sigma}}.$$

Sufficiently high rebates. Because $s_{11} - s_{21} \geq 0$,

$$\frac{1}{2\sigma(\sigma - 1)(1 + \chi)} > 0, \quad \frac{(s_{11} - s_{21})[(\sigma - 1)^2 + \sigma^2\chi]}{2\sigma(\sigma - 1)[1 + \chi(s_{11} - s_{21})^2]} \geq 0.$$

Therefore $\lambda^{\tau_{LB}}/s_{21} < \sigma/[2(\sigma - 1)] - \rho$. If $\sigma \geq 2$ and $\rho \geq \sigma/[2(\sigma - 1)]$, the multiplier is negative, excluding free trade. Higher rebates increase the return from taxing the existing import base.

Sufficiently elastic entry. Dropping $-\rho$ and $-1/[2\sigma(\sigma - 1)(1 + \chi)]$ shows that free trade is not optimal if

$$\sigma^2\chi(s_{11} - s_{21})[1 - (s_{11} - s_{21})] > \sigma^2 - (\sigma - 1)^2(s_{11} - s_{21}).$$

The sufficient bound in the lemma,

$$\chi \geq \frac{(1 + \theta^{1-\sigma})^2}{2\theta^{1-\sigma}(1 - \theta^{1-\sigma})},$$

makes the left side at least σ^2 . The right side is strictly smaller than σ^2 because $\theta > 1$. Under this sufficient condition, protection is optimal at every rebate rate, including zero. Responsive entry makes promised protection valuable through its effect on firm location.

Under either condition, the lower-bound multiplier that free trade would require is negative, which is not allowed. Since Appendix B.8.4 shows that the stationary tariff is finite, $1 < \tau_{21} < \infty$ for every regular convergent optimal plan.

B.8.4 Bound on the stationary tariff

In this section, we show that the stationary tariff in Lemma 7 is finite. Maintain $\sigma > 1$, finite θ , fixed finite $\chi > 0, \rho \in [0, 1], \tau_{12} = 1$, positive finite limiting stocks, strictly unsaturated entry, finite multiplier limits, and the regularity assumptions maintained in Lemma 7.

The limiting private allocation and multipliers. As $\tau_{21} \rightarrow \infty$, (A.15) becomes

$$\left(\frac{n_1}{n_2}\right)^{1+1/\chi} = 1 + 2\theta^{1-\sigma} \frac{n_1}{n_2}.$$

Its left side minus its right side is strictly convex in n_1/n_2 , negative at $n_1/n_2 = 0$, and tends to infinity as $n_1/n_2 \rightarrow \infty$. This equation therefore has a unique positive finite solution. Thus $s_{11} \rightarrow 1$, $s_{21} \rightarrow 0$, and $0 < s_{12} < 1$, where s_{12} below denotes the limit of s_{12} .

For $\rho = 0$, the stationary versions of (A.3), (A.4), (A.12) and (A.13) have the limits

$$\begin{aligned} \lambda_1^P P_1^{1-\sigma} &= \frac{1}{\sigma(\sigma-1)} - \frac{1}{\sigma} \frac{\lambda_1^v}{n_1}, \\ \lambda_2^P P_2^{1-\sigma} &= -\frac{1}{\sigma} \left[\left(1 + \frac{\lambda_1^v}{n_1}\right) s_{12} + \frac{\lambda_2^v}{n_2} (1 - s_{12}) \right], \\ \frac{\lambda_1^v}{n_1} &= \frac{\chi^\sigma}{1 + s_{12}} \left[\lambda_1^P P_1^{1-\sigma} + s_{12} \lambda_2^P P_2^{1-\sigma} \right], \\ \frac{\lambda_2^v}{n_2} &= \chi^\sigma \lambda_2^P P_2^{1-\sigma}. \end{aligned}$$

The second and fourth equations above give $\lambda_2^P P_2^{1-\sigma} = -s_{12}(1 + \lambda_1^v/n_1)/\{\sigma[1 + \chi(1 - s_{12})]\}$. Substituting this and the first equation into the third gives

$$\frac{\lambda_1^v}{n_1} = \frac{\chi[1 + \chi(1 - s_{12}) - (\sigma - 1)s_{12}^2]}{(\sigma - 1)(1 + \chi)[1 + s_{12} + \chi(1 - s_{12})]}. \quad (\text{B.12})$$

Hence

$$\frac{\lambda_2^v}{n_2} = -\frac{\chi s_{12}}{1 + \chi(1 - s_{12})} \left(1 + \frac{\lambda_1^v}{n_1}\right) = -\frac{\chi s_{12}[(\sigma - 1)(1 + s_{12}) + \sigma\chi]}{(\sigma - 1)(1 + \chi)[1 + s_{12} + \chi(1 - s_{12})]} < 0. \quad (\text{B.13})$$

Substituting (B.12) into the first equation above gives

$$\lambda_1^P P_1^{1-\sigma} = \frac{1 + s_{12} + \chi[1 + (\sigma - 1)s_{12}^2]}{\sigma(\sigma - 1)(1 + \chi)[1 + s_{12} + \chi(1 - s_{12})]} > 0.$$

With $\rho > 0$, the limits of (A.3) and (A.12) become

$$\begin{aligned} \lambda_1^P P_1^{1-\sigma} - \rho &= \frac{1}{\sigma(\sigma - 1)} - \frac{1}{\sigma} \frac{\lambda_1^v}{n_1}, \\ \frac{\lambda_1^v}{n_1} &= \frac{\chi^\sigma}{1 + s_{12}} \left[\lambda_1^P P_1^{1-\sigma} - \rho + s_{12} \lambda_2^P P_2^{1-\sigma} \right], \end{aligned}$$

while the limits of (A.4) and (A.13) do not contain ρ . These are the limits for $\rho = 0$ with $\lambda_1^P P_1^{1-\sigma}$ replaced by $\lambda_1^P P_1^{1-\sigma} - \rho$. Hence λ_1^v/n_1 and λ_2^v/n_2 keep the limits in (B.12) and (B.13), and $\lambda_1^P P_1^{1-\sigma}$ rises by ρ . The right side of (A.14) therefore has the limit

$$\lim_{\tau_{21} \rightarrow \infty} \left[(\sigma - 1) \lambda_1^P P_1^{1-\sigma} + \frac{\lambda_2^v/n_2 - \rho}{\tau_{21}} \right] = (\sigma - 1) \rho + \frac{1 + s_{12} + \chi[1 + (\sigma - 1)s_{12}^2]}{\sigma(1 + \chi)[1 + s_{12} + \chi(1 - s_{12})]} > 0. \quad (\text{B.14})$$

An interior stationary tariff requires the right side of (A.14) to be zero. By (B.14), it is instead strictly positive at all sufficiently high finite tariffs, so the stationary tariff cannot diverge to infinity.

Intuitively, the marginal loss through the home price index is proportional to s_{21}/τ_{21} , whereas the gain from further depressing foreign profits is proportional to s_{21}/τ_{21}^2 . λ_1^v/n_1 and λ_2^v/n_2 have finite limits, so the latter gain vanishes faster. At fixed finite entry elasticity, raising an already sufficiently high tariff lowers welfare under the maintained regularity assumptions.

B.9 Proof of Proposition 4

The initial tariff is characterized in proofs of Proposition 3 and Lemma 6, and the long-run protection in the proof of Lemma 7. The analysis below derives the precise sufficient parameter restrictions for higher long-run protection, lower long-run protection, and stationary free trade. Stationary stocks remain endogenous in all positive-tariff comparisons. The common stationary equations are derived in Appendix A.3.

B.9.1 Part (i): Higher protection after initial free trade

For part (i) of Proposition 4, suppose

$$\rho \leq \frac{1/\sigma + \theta^{1-\sigma}}{1 + \theta^{1-\sigma}}, \quad \chi \geq \frac{(1 + \theta^{1-\sigma})^2}{2\theta^{1-\sigma}(1 - \theta^{1-\sigma})}.$$

Lemma 6 implies initial free trade, while part (ii) of Lemma 7 excludes stationary free trade at every rebate rate. Its finite-tariff conclusion yields $1 = \tau_{21,0} < \tau_{21} < \infty$. Promised protection induces entry that the current tariff cannot affect.

B.9.2 Extension: Higher protection after positive initial protection

The higher-protection extension stated in the footnote to part (i) of Proposition 4 also allows a positive initial tariff. Fix $\sigma > 1$, $\rho \in (1/\sigma, 1]$, and finite $\chi > 0$. An explicit sufficient restriction is

$$\theta^{\sigma-1} > \max \left\{ \frac{1-\rho}{\rho-1/\sigma}, \sigma[1 + (\sigma-1)\rho] \left(1 + \frac{1}{\chi} \right) + 8\sigma \left(5 + \frac{6}{\sigma-1} + \sigma\rho \right) \right\}. \quad (\text{B.15})$$

The first term inside the maximum is equivalent to the strict initial-protection condition in Lemma 6. The second is equivalent to

$$\theta^{1-\sigma} \left[1 + (\sigma-1)\rho + \frac{8\chi}{1+\chi} \left(5 + \frac{6}{\sigma-1} + \sigma\rho \right) \right] < \frac{\chi}{\sigma(1+\chi)}. \quad (\text{B.16})$$

For each fixed σ, ρ, χ in this range, these bounds hold for sufficiently large finite θ . The latter restriction also implies $\theta^{1-\sigma} < 1/16$, so the uniform bound (B.10) applies.

The limit of the right side of (A.14) as $\theta^{1-\sigma} \rightarrow 0$, $(\sigma-1)\rho + 1/[\sigma(1+\chi)] - \sigma\rho/\tau_{21}$, increases with τ_{21} . At $\tau_{21} = \sigma^2\rho/[1 + \sigma(\sigma-1)\rho]$, the upper bound on $\tau_{21,0}$ in the proof of Lemma 6, it equals

$$(\sigma-1)\rho + \frac{1}{\sigma(1+\chi)} - \frac{\sigma\rho}{\sigma^2\rho/[1 + \sigma(\sigma-1)\rho]} = -\frac{\chi}{\sigma(1+\chi)}.$$

It is at most this strictly negative number for every $1 \leq \tau_{21} \leq \sigma^2\rho/[1 + \sigma(\sigma-1)\rho]$. By (B.10) and (B.16), the right side of (A.14) differs from this limit by less than $\chi/[\sigma(1+\chi)]$. Therefore the right side of (A.14) is negative throughout that interval. This implies free trade is not optimal, because free trade requires a nonnegative lower-bound multiplier. It also rules out positive interior stationary tariffs in this interval, which require the right side of (A.14) to be zero.

Every regular stationary limit must consequently lie above $\sigma^2\rho/[1 + \sigma(\sigma-1)\rho]$, which exceeds $\tau_{21,0}$, giving $1 < \tau_{21,0} < \tau_{21} < \infty$. High trade costs make supplying varieties domestically valuable, and the entry response to promised protection adds a gain unavailable to the initial tariff.

B.9.3 Part (ii): Lower long-run protection

Fix $\sigma > 1$ and maintain the symmetric initial stocks and the existence, convergence, and regularity assumptions. For sufficiently low trade costs and sufficiently small positive entry elasticity,

$$\frac{1/\sigma + \theta^{1-\sigma}}{1 + \theta^{1-\sigma}} < \rho \leq \min \left\{ 1, \frac{\sigma+1}{2\sigma} + \frac{1}{4\sigma^3} \right\}$$

implies $1 \leq \tau_{21} < \tau_{21,0}$. The restrictions on trade costs and entry elasticity can be chosen uniformly over this rebate interval. The proof first evaluates the effect of entry at $\theta = 1$ and $\chi = 0$, then extends the comparison to positive trade costs and entry elasticity. Zero entry elasticity is used only as a limit. For the calculation at $\theta = 1$ and $\chi = 0$, fix rebates satisfying

$$\frac{\sigma+1}{2\sigma} < \rho \leq \min \left\{ 1, \frac{\sigma+1}{2\sigma} + \frac{1}{4\sigma^3} \right\}. \quad (\text{B.17})$$

Initial protection. At $\theta = 1$ and symmetric initial stocks, the proof of Lemma 6 gives $\tau_{21,0} > 1$ exactly when $\rho > (\sigma+1)/(2\sigma)$, which holds under (B.17). Then $\tau_{21,0}$ satisfies the first-order

condition

$$\rho = \frac{\tau_{21,0}[1 - (\sigma - 1)s_{11,0}/\sigma]}{1 - (\sigma - 1)(\tau_{21,0} - 1)s_{11,0}}, \quad s_{11,0} = \frac{\tau_{21,0}^{\sigma-1}}{1 + \tau_{21,0}^{\sigma-1}}.$$

For every tariff between one and $\tau_{21,0}$, the denominator in this condition is positive. On this interval, the derivative of the right side of this condition with respect to the tariff is at least one, by (B.3). At free trade, this right side equals $(\sigma + 1)/(2\sigma)$, so

$$0 < \tau_{21,0} - 1 \leq \rho - \frac{\sigma + 1}{2\sigma} \leq \frac{1}{4\sigma^3}.$$

Since $ds_{11,0}/d\tau_{21,0} \leq (\sigma - 1)/4$, integration also gives

$$\begin{aligned} 0 < s_{11,0} - \frac{1}{2} &\leq \frac{\sigma - 1}{4} \left(\rho - \frac{\sigma + 1}{2\sigma} \right) \leq \frac{\sigma - 1}{16\sigma^3} < \frac{1}{4}, \\ \frac{1}{2} &\leq s_{11,0} \leq \frac{3}{4}, \quad \frac{1}{4} \leq s_{21,0} \leq \frac{1}{2}. \end{aligned} \tag{B.18}$$

Entry and the stationary tariff. At $\chi = 0$, the limiting private stock ratio n_1/n_2 is one and $\lambda_1^v/n_1 = \lambda_2^v/n_2 = 0$. Setting the right side of (A.14) to zero then gives the same equation as the first-order condition for $\tau_{21,0}$ above. Hence, at the stationary tariff,

$$\begin{aligned} \tau_{21} &= \tau_{21,0}, \quad s_{11} = s_{11,0}, \quad s_{21} = s_{21,0}, \quad s_{12} = s_{22} = \frac{1}{2}, \\ \lambda_i^v/n_i &= 0, \quad \lambda_1^p P_1^{1-\sigma} = \frac{\rho}{(\sigma - 1)\tau_{21,0}}, \quad \lambda_2^p P_2^{1-\sigma} = -\frac{1}{2\sigma}. \end{aligned}$$

In the following derivatives, hold σ and ρ fixed, set $\theta = 1$, and keep the tariff at $\tau_{21} = \tau_{21,0}$, allowing private stocks and multipliers to respond to χ . Because the right sides of (A.12)–(A.13) are proportional to χ , differentiation at zero gives

$$\begin{aligned} \left. \frac{\partial(\lambda_1^v/n_1)}{\partial\chi} \right|_{\chi=0} &= \frac{\sigma s_{11}\rho[\sigma/((\sigma - 1)\tau_{21,0}) - 1] - 1/4}{s_{11} + 1/2}, \\ \left. \frac{\partial(\lambda_2^v/n_2)}{\partial\chi} \right|_{\chi=0} &= \frac{\sigma s_{21}\rho/((\sigma - 1)\tau_{21,0}) - 1/4}{1/2 + s_{21}/\tau_{21,0}}. \end{aligned} \tag{B.19}$$

Subtracting $1/2$ from the first derivative in (B.19), scaled by $\sigma - 1$, gives

$$\begin{aligned} \left(s_{11} + \frac{1}{2} \right) \left[(\sigma - 1) \left. \frac{\partial(\lambda_1^v/n_1)}{\partial\chi} \right|_{\chi=0} - \frac{1}{2} \right] &= \left(\sigma\rho - \frac{1}{2} \right) \left(s_{11} - \frac{1}{2} \right) + \frac{\sigma}{2} \left(\rho - \frac{\sigma + 1}{2\sigma} \right) \\ &\quad - \sigma^2 s_{11}\rho \frac{\tau_{21,0} - 1}{\tau_{21,0}}. \end{aligned}$$

On the right, the first two terms are nonnegative and the last term is negative. By the bounds on $\tau_{21,0}$ and the shares above, the first two terms sum to at most $\frac{1}{2}\sigma^2[\rho - (\sigma + 1)/(2\sigma)]$, and the last term is no smaller than $-\frac{3}{4}\sigma^2[\rho - (\sigma + 1)/(2\sigma)]$. The absolute value of the right side is therefore

at most $\frac{3}{4}\sigma^2[\rho - (\sigma + 1)/(2\sigma)]$. Since $s_{11} + 1/2 \geq 1$,

$$\left| (\sigma - 1) \frac{\partial(\lambda_1^v/n_1)}{\partial\chi} \Big|_{\chi=0} - \frac{1}{2} \right| \leq \frac{3}{4}\sigma^2 \left(\rho - \frac{\sigma + 1}{2\sigma} \right). \quad (\text{B.20})$$

For the second derivative, the corresponding identity is

$$\left(\frac{\tau_{21,0}}{2} + s_{21} \right) \left[(\sigma - 1) \frac{\partial(\lambda_2^v/n_2)}{\partial\chi} \Big|_{\chi=0} - \frac{1}{2} \right] = \sigma \left[s_{21} \left(\rho - \frac{\sigma + 1}{2\sigma} \right) - \frac{s_{11} - 1/2}{2} - \frac{\tau_{21,0} - 1}{4} \right].$$

The same bounds give an absolute value no greater than $\frac{1}{2}\sigma^2[\rho - (\sigma + 1)/(2\sigma)]$ on the right. Dividing by $\tau_{21,0}/2 + s_{21} \geq 3/4$ yields

$$\left| (\sigma - 1) \frac{\partial(\lambda_2^v/n_2)}{\partial\chi} \Big|_{\chi=0} - \frac{1}{2} \right| \leq \frac{2}{3}\sigma^2 \left(\rho - \frac{\sigma + 1}{2\sigma} \right). \quad (\text{B.21})$$

To obtain the derivative of s_{11} with respect to χ , differentiate $(1 + \chi) \ln(n_1/n_2) = \chi \ln[(s_{11} + s_{12})/(s_{22} + s_{21}/\tau_{21,0})]$ at zero:

$$\frac{\partial s_{11}}{\partial\chi} \Big|_{\chi=0} = s_{11}s_{21} \ln \frac{s_{11} + 1/2}{1/2 + s_{21}/\tau_{21,0}}.$$

Differentiate the right side of (A.14) with respect to χ at $\tau_{21} = \tau_{21,0}$, using the derivative of s_{11} just obtained and $(\sigma - 1)\lambda_1^p P_1^{1-\sigma} = \rho/\tau_{21,0}$ from above:

$$\begin{aligned} & \sigma(\sigma - 1) \frac{\partial}{\partial\chi} \left[(\sigma - 1)\lambda_1^p P_1^{1-\sigma} + \frac{\lambda_2^v/n_2 - \rho}{\tau_{21,0}} \right] \Big|_{\chi=0} \\ &= \sigma(\sigma - 1) \left(\frac{\rho}{\tau_{21,0}} - 1 \right) s_{21} \ln \left(\frac{s_{11} + 1/2}{1/2 + s_{21}/\tau_{21,0}} \right) \\ &+ \frac{1 + (\sigma - 1)s_{11}}{\tau_{21,0}} (\sigma - 1) \frac{\partial(\lambda_2^v/n_2)}{\partial\chi} \Big|_{\chi=0} \\ &- (\sigma - 1)s_{11}(\sigma - 1) \frac{\partial(\lambda_1^v/n_1)}{\partial\chi} \Big|_{\chi=0}. \end{aligned} \quad (\text{B.22})$$

The last two terms on the right of (B.22), which contain the derivatives of λ_1^v/n_1 and λ_2^v/n_2 , are bounded below by

$$\frac{1}{2} - \left(\frac{\sigma}{2} + \frac{17\sigma^3}{16} \right) \left(\rho - \frac{\sigma + 1}{2\sigma} \right).$$

This follows from (B.20)–(B.21), $s_{11} \leq 3/4$, and $1 + (\sigma - 1)s_{11} \leq \sigma$. The first term on the right of (B.22), which contains the derivative of s_{11} , is negative. Using $\ln x \leq x - 1$, the bounds on $\tau_{21,0}$ and the shares above give

$$0 < \ln \frac{s_{11} + 1/2}{1/2 + s_{21}/\tau_{21,0}} \leq \sigma \left(\rho - \frac{\sigma + 1}{2\sigma} \right), \quad 0 < \frac{\tau_{21,0} - \rho}{\tau_{21,0}} \leq \frac{\sigma - 1}{2\sigma}.$$

This first term is therefore no smaller than $-\sigma(\sigma - 1)^2[\rho - (\sigma + 1)/(2\sigma)]/4$. Adding the two lower

bounds gives

$$\begin{aligned}
& \sigma(\sigma-1) \frac{\partial}{\partial \chi} \left[(\sigma-1) \lambda_1^p P_1^{1-\sigma} + \frac{\lambda_2^v/n_2 - \rho}{\tau_{21,0}} \right] \Big|_{\chi=0} \\
& \geq \frac{1}{2} - \left[\frac{\sigma}{2} + \frac{17}{16} \sigma^3 + \frac{\sigma(\sigma-1)^2}{4} \right] \left(\rho - \frac{\sigma+1}{2\sigma} \right) \\
& \geq \frac{1}{2} - \frac{29}{16} \sigma^3 \left(\rho - \frac{\sigma+1}{2\sigma} \right) \geq \frac{3}{64} > 0.
\end{aligned} \tag{B.23}$$

Lower long-run protection. At $\chi = 0$, the right-hand side of (A.14) is

$$1 - \frac{\rho}{\tau_{21}} + \frac{(\sigma-1)[\rho(1 - \tau_{21}^{-1}) - 1/\sigma]}{1 + \theta^{1-\sigma} \tau_{21}^{1-\sigma}}.$$

Its unique zero is the initial optimum, and it is positive at every higher tariff. At that zero, the derivative of this expression with respect to τ_{21} is

$$\frac{1 + (\sigma-1)^2(\tau_{21,0} - 1)s_{11}s_{21}}{\tau_{21,0}[1 - (\sigma-1)(\tau_{21,0} - 1)s_{11}]} > 0.$$

Together with (B.23), implicit differentiation gives $d\tau_{21}/d\chi|_{\chi=0} < 0$ at $\theta = 1$. The initial tariff does not depend on χ .

This comparison extends to the entire rebate interval stated above. At $\rho = (\sigma+1)/(2\sigma)$ and $\theta = 1$, the initial tariff is one, and (B.19) and (B.22) imply that, at $\chi = 0$ and $\tau_{21} = 1$, the derivative of the right side of (A.14) with respect to χ is $1/[2\sigma(\sigma-1)] > 0$. Hence the lower bound in (B.23) holds for every rebate in (B.17) and also at $\rho = (\sigma+1)/(2\sigma)$. Near $\chi = 0$, the stationary stocks and multipliers are smooth functions of χ : at $\chi = 0$, the derivative of $(1+\chi) \ln(n_1/n_2) - \chi \ln[(s_{11} + s_{12})/(s_{22} + s_{21}/\tau_{21})]$ with respect to $\ln(n_1/n_2)$ is one, and (B.7)–(B.8) reduce to $\lambda_1^v/n_1 = 0$ and $\lambda_2^v/n_2 = 0$. Continuity over the closed rebate interval, together with the continuity of the initial tariff in trade costs and rebates, keeps the derivative of the right side of (A.14) with respect to χ at $\tau_{21} = \tau_{21,0}$ positive for sufficiently low trade costs and entry elasticity. Rebates between $(1/\sigma + \theta^{1-\sigma})/(1 + \theta^{1-\sigma})$, the lower end of the rebate interval in the statement, and $(\sigma+1)/(2\sigma)$ approach $(\sigma+1)/(2\sigma)$ as $\theta \rightarrow 1$ and are covered by the same argument. Integrating from $\chi = 0$ shows that the right side of (A.14) is strictly positive at $\tau_{21} = \tau_{21,0}$.

The derivative of the right side of (A.14) with respect to τ_{21} remains positive near these initial optima, including at free trade, where the derivative displayed above equals one. The right side of (A.14) is consequently positive just above the initial tariff. At higher tariffs outside these neighborhoods, its $\chi = 0$ value is bounded away from zero, including the limit $1/\sigma + (\sigma-1)\rho > 0$ as $\tau_{21} \rightarrow \infty$. Since $n_1/n_2 \geq 1$ by (A.16), bounding the denominator of (A.15) below by n_1/n_2 gives $(n_1/n_2)^{1/\chi} \leq (1 + \theta^{2(1-\sigma)})/(n_1/n_2) + 2\theta^{1-\sigma} \leq (1 + \theta^{1-\sigma})^2 \leq 4$. The bound $1 \leq n_1/n_2 \leq 4^\chi$ and bounded coefficients in the multiplier equations make the right side of (A.14) converge, as $\chi \rightarrow 0$, to its value at $\chi = 0$ given above, uniformly in $1/\tau_{21} \in [0, 1]$. Thus, for sufficiently small positive χ , no finite stationary tariff at or above $\tau_{21,0}$ satisfies the condition that the right side of (A.14) be zero. Appendix B.8.4 rules out tariffs diverging to infinity. Every regular optimal limit therefore satisfies $1 \leq \tau_{21} < \tau_{21,0}$.

Now fix a rebate strictly above $(\sigma+1)/(2\sigma)$. At $\chi = 0$, the right side of (A.14) is negative at free trade and has exactly one zero above one. For small positive χ , the right side of (A.14) is uniformly close to its value at $\chi = 0$, so it has no zero away from that point; near that point, its derivative with

respect to τ_{21} is positive, so it has exactly one zero there. These signs persist for sufficiently low positive trade costs and entry elasticity, giving $1 < \tau_{21} < \tau_{21,0}$. Only local derivative convergence near the finite zero is needed, so this argument also applies when $1 < \sigma < 2$. Low trade costs make foreign varieties valuable, and the incentive to encourage their entry reduces protection promised for the long run.

B.9.4 Stationary free trade after initial protection

Maintain the assumptions of Proposition 4, namely those stated before Lemma 6. The following sufficient conditions imply $1 = \tau_{21} < \tau_{21,0}$:

$$\begin{aligned} 0 < \chi &\leq \frac{\sigma - 1}{200\sigma^2}, \\ 1 < \theta &\leq \left[\frac{2\sigma^2(1 + \chi) + 1}{2\sigma^2(1 + \chi) - 1} \right]^{1/(\sigma-1)}, \\ \frac{1/\sigma + \theta^{1-\sigma}}{1 + \theta^{1-\sigma}} < \rho &\leq \frac{1/\sigma + \theta^{1-\sigma}}{1 + \theta^{1-\sigma}} + \frac{\chi}{4\sigma(\sigma - 1)(1 + \chi)}, \\ \rho &\leq 1. \end{aligned} \tag{B.24}$$

The set of parameters satisfying (B.24) is nonempty for every $\sigma > 1$. The strict lower bound on ρ in (B.24) is the condition in the proof of Lemma 6 for $\tau_{21,0} > 1$ at symmetric initial stocks, so the initial tariff is positive. It remains to rule out protection in the stationary equilibrium.

By (A.16), $n_1/n_2 \geq 1$, and Appendix B.9.3 shows that $(n_1/n_2)^{1/\chi} \leq 4$; hence $1 \leq n_1/n_2 \leq 4^\chi < 2$. The right side of (A.15) does not increase with n_1/n_2 , and at $n_1/n_2 = 1$ it exceeds one by at most $2\sigma(1 - \tau_{21}^{-1})$. Taking logs in (A.15) and using $\ln x \leq x - 1$ therefore yields

$$0 \leq \ln \frac{n_1}{n_2} \leq 2\sigma\chi(1 - \tau_{21}^{-1}). \tag{B.25}$$

The derivatives of s_{11} and s_{12} with respect to $\ln(n_1/n_2)$ are $s_{11}s_{21}$ and $s_{12}s_{22}$, both at most $1/4$. Hence, compared with equal stocks at the same tariff, the higher stock ratio raises s_{11} by at most $\sigma\chi(1 - \tau_{21}^{-1})/2$. Compared with free trade, where $n_1 = n_2$, the shares satisfy

$$\max_{ij} |s_{ij} - s_{ij}|_{\tau_{21}=1} \leq \sigma(1 - \tau_{21}^{-1}).$$

Now consider (B.7)–(B.8), which are linear in λ_1^v/n_1 and λ_2^v/n_2 (Appendix B.8.2). In (B.7), the coefficients of λ_1^v/n_1 and λ_2^v/n_2 are $1 + \chi(s_{11}^2 + s_{12}^2)/(s_{11} + s_{12})$ and $\chi(s_{11}s_{21}/\tau_{21} + s_{12}s_{22})/(s_{11} + s_{12})$; in (B.8), they are $\chi(s_{21}s_{11} + s_{22}s_{12})/(s_{22} + s_{21}/\tau_{21})$ and $1 + \chi(s_{21}^2/\tau_{21} + s_{22}^2)/(s_{22} + s_{21}/\tau_{21})$. Because $s_{11} + s_{21} = s_{12} + s_{22} = 1$, $s_{21} + s_{22} \leq 1$ (as $n_1/n_2 \geq 1$ and $\tau_{21} \geq 1$), and $s_{22} \geq 1/(1 + 4^\chi) > 1/3$, the fractions multiplying χ satisfy

$$\begin{aligned} \frac{s_{11}^2 + s_{12}^2}{s_{11} + s_{12}} + \frac{s_{11}s_{21}/\tau_{21} + s_{12}s_{22}}{s_{11} + s_{12}} &\leq 1, \\ \frac{s_{21}s_{11} + s_{22}s_{12}}{s_{22} + s_{21}/\tau_{21}} + \frac{s_{21}^2/\tau_{21} + s_{22}^2}{s_{22} + s_{21}/\tau_{21}} &\leq \frac{1}{s_{22}} < 3. \end{aligned}$$

Hence, if $|\lambda_1^v/n_1| \geq |\lambda_2^v/n_2|$, the left side of (B.7) is at least $(1 - \chi)|\lambda_1^v/n_1|$ in absolute value, and if $|\lambda_2^v/n_2| \geq |\lambda_1^v/n_1|$, the left side of (B.8) is at least $(1 - 3\chi)|\lambda_2^v/n_2|$. The larger of $|\lambda_1^v/n_1|$ and

$|\lambda_2^v/n_2|$ is therefore at most $(1 - 3\chi)^{-1} \leq 2$ times the larger absolute value of the two right sides. At free trade, $s_{11} = s_{22}$, $s_{12} = s_{21}$, and $\tau_{21} = 1$, so both sums in this display equal one and the factor $(1 - 3\chi)^{-1}$ improves to $(1 - \chi)^{-1}$. Also at free trade, the terms with ρ on the right sides of (B.7)–(B.8) vanish, $\sigma s_{11}/[(\sigma - 1)(s_{11} + s_{12})]$ and $\sigma s_{21}/[(\sigma - 1)(s_{22} + s_{21}/\tau_{21})]$ are at most $\sigma/(\sigma - 1)$, and $(s_{11}^2 + s_{12}^2)/(s_{11} + s_{12})$ and $(s_{21}s_{11} + s_{22}s_{12})/(s_{22} + s_{21}/\tau_{21})$ lie between zero and one, so each right side is at most $2\chi\sigma/(\sigma - 1)$ in absolute value. This gives

$$\max_{i=1,2} \left| \frac{\lambda_i^v}{n_i} \right|_{\tau_{21}=1} \leq \frac{2\chi\sigma}{(\sigma - 1)(1 - \chi)} \leq \frac{3\chi\sigma}{\sigma - 1}. \quad (\text{B.26})$$

Next, compare the four coefficients listed above and the right sides of (B.7)–(B.8) with their free-trade values. Every share and τ_{21}^{-1} lies between zero and one and changes by at most $\sigma(1 - \tau_{21}^{-1})$ relative to free trade, so a product of two of these terms changes by at most $2\sigma(1 - \tau_{21}^{-1})$ and a product of three by at most $3\sigma(1 - \tau_{21}^{-1})$. Moreover, a fraction changes by at most the change in its numerator plus its free-trade value times the change in its denominator, divided by its denominator. The denominators $s_{11} + s_{12}$ and $s_{22} + s_{21}/\tau_{21}$ are at least $s_{11} \geq 1/2$ and $s_{22} \geq 1/(1 + 4\chi) > 0.49$, and they change by at most $2\sigma(1 - \tau_{21}^{-1})$ and $3\sigma(1 - \tau_{21}^{-1})$. The numerators, $s_{11}^2 + s_{12}^2$ and $s_{11}s_{21}/\tau_{21} + s_{12}s_{22}$ in (B.7) and $s_{21}s_{11} + s_{22}s_{12}$ and $s_{21}^2/\tau_{21} + s_{22}^2$ in (B.8), change by at most $4\sigma(1 - \tau_{21}^{-1})$ and $5\sigma(1 - \tau_{21}^{-1})$, respectively. Each of the four fractions multiplying χ is at most one at free trade, so in each equation the two of them together change by at most $(9 + 2 \cdot 3)\sigma(1 - \tau_{21}^{-1})/0.49 < 32\sigma(1 - \tau_{21}^{-1})$. On the right sides, the same rule bounds the changes in $s_{11}/(s_{11} + s_{12})$ and $s_{21}/(s_{22} + s_{21}/\tau_{21})$ by $6\sigma(1 - \tau_{21}^{-1})$, and the changes in $(s_{11}^2 + s_{12}^2)/(s_{11} + s_{12})$ and $(s_{21}s_{11} + s_{22}s_{12})/(s_{22} + s_{21}/\tau_{21})$ by $12\sigma(1 - \tau_{21}^{-1})$; for the two fractions in (B.8) this uses their free-trade values s_{21} and $2s_{11}s_{21}$, which are at most $1/2$. The terms with ρ vanish at free trade and are otherwise at most $\sigma(1 - \tau_{21}^{-1})$ in absolute value. Since $\sigma < \sigma^2/(\sigma - 1)$, each right side changes by at most $\chi(6 + 12 + 1)\sigma^2(1 - \tau_{21}^{-1})/(\sigma - 1) < 20\chi\sigma^2(1 - \tau_{21}^{-1})/(\sigma - 1)$. Subtracting the free-trade equations from (B.7)–(B.8) leaves on the right the changes in the right sides minus the coefficient changes times the free-trade values of λ_1^v/n_1 and λ_2^v/n_2 . Applying the bound above and (B.26) therefore gives

$$\max_{i=1,2} \left| \frac{\lambda_i^v}{n_i} - \frac{\lambda_i^v}{n_i} \right|_{\tau_{21}=1} \leq 2\chi \left[\frac{20\sigma^2}{\sigma - 1} + 32\sigma \frac{3\chi\sigma}{\sigma - 1} \right] (1 - \tau_{21}^{-1}) \leq \frac{72\chi\sigma^2}{\sigma - 1} (1 - \tau_{21}^{-1}).$$

Next, compare the right side of (A.14) at a tariff $\tau_{21} > 1$ with its value at free trade; the left side of (B.27) is this difference. If $\chi = 0$ and ρ equals the lower bound $(1/\sigma + \theta^{1-\sigma})/(1 + \theta^{1-\sigma})$ in (B.24), the convexity inequality $\tau_{21}^{-\sigma} \geq 1 + \sigma(\tau_{21}^{-1} - 1)$ shows that the difference is at least $1 - \tau_{21}^{-1}$. A higher ρ raises the difference. For $\chi > 0$, three effects can lower it. First, because $n_1/n_2 > 1$ at $\tau_{21} > 1$, s_{11} is higher than with equal stocks; after substituting (A.3) for $\lambda_1^P P_1^{1-\sigma}$, this lowers the difference by at most $(\sigma - 1)\chi(1 - \tau_{21}^{-1})/2$. Second, after this substitution, λ_1^v/n_1 and λ_2^v/n_2 enter the right side of (A.14) with coefficients of absolute value at most one, so their changes relative to free trade lower the difference by at most $144\chi\sigma^2(1 - \tau_{21}^{-1})/(\sigma - 1)$. Third, these two coefficients change relative to free trade by at most $(\sigma - 1)(1 - \tau_{21}^{-1})$ and $\sigma(1 - \tau_{21}^{-1})$; since (B.26) bounds the free-trade values of λ_1^v/n_1 and λ_2^v/n_2 , this lowers the difference by at most $6\chi\sigma^2(1 - \tau_{21}^{-1})/(\sigma - 1)$.

Adding these bounds gives

$$\begin{aligned}
& (\sigma - 1)\lambda_1^P P_1^{1-\sigma} + \frac{\lambda_2^V/n_2 - \rho}{\tau_{21}} - \left[(\sigma - 1)\lambda_1^P P_1^{1-\sigma} + \frac{\lambda_2^V/n_2 - \rho}{\tau_{21}} \right] \Big|_{\tau_{21}=1} \\
& \geq (1 - \tau_{21}^{-1}) \left[1 - \frac{(\sigma - 1)\chi}{2} - \frac{150\chi\sigma^2}{\sigma - 1} \right] \geq (1 - \tau_{21}^{-1}) \left[1 - \frac{151\chi\sigma^2}{\sigma - 1} \right].
\end{aligned} \tag{B.27}$$

Under (B.24), $\chi \leq (\sigma - 1)/(200\sigma^2)$, so $1 - 151\chi\sigma^2/(\sigma - 1) \geq 49/200 > 0$.

It remains to check free trade itself. At free trade, $n_1 = n_2$ and $s_{11} - s_{21} = (1 - \theta^{1-\sigma})/(1 + \theta^{1-\sigma})$. By (B.11), the lower-bound multiplier at free trade is nonnegative exactly when ρ is at most the sum of the two fractions inside the brackets of (B.11). Subtracting from this sum the lower bound on ρ in (B.24), which at free trade equals $1 - (\sigma - 1)s_{11}/\sigma$, gives

$$\begin{aligned}
& \frac{\sigma - 1 + \sigma\chi}{2\sigma(\sigma - 1)(1 + \chi)} + \frac{[\sigma - 1 - \sigma\chi(s_{11} - s_{21})][\sigma - (\sigma - 1)(s_{11} - s_{21})]}{2\sigma(\sigma - 1)[1 + \chi(s_{11} - s_{21})^2]} - \left[1 - \frac{\sigma - 1}{\sigma}s_{11} \right] \\
& = \frac{\chi}{2\sigma(\sigma - 1)} \left[\frac{1}{1 + \chi} - \frac{(s_{11} - s_{21})[\sigma^2 - (\sigma - 1)^2(s_{11} - s_{21})^2]}{1 + \chi(s_{11} - s_{21})^2} \right].
\end{aligned} \tag{B.28}$$

The upper bound on θ in (B.24) is equivalent to $s_{11} - s_{21} \leq 1/[2\sigma^2(1 + \chi)]$, and $\theta > 1$ gives $s_{11} - s_{21} > 0$, so the right side of (B.28) is at least $\chi/[4\sigma(\sigma - 1)(1 + \chi)]$. The upper bound on ρ in (B.24) exceeds the lower bound by exactly this amount, so ρ does not exceed the sum of the two fractions inside the brackets of (B.11), and the lower-bound multiplier at free trade is nonnegative; equivalently, the right side of (A.14) is nonnegative at free trade. By (B.27), the right side of (A.14) is then strictly positive at every finite tariff above one, so no such tariff satisfies (A.14) with $\lambda^{\tau_{LB}} = 0$, while Appendix B.8.4 rules out tariffs diverging to infinity. Thus every regular convergent optimum has $\tau_{21} = 1 < \tau_{21,0}$: the fiscal gain justifies taxing existing imports, but the cost of discouraging future foreign entry makes long-run free trade preferable.

C Numerical solution of the commitment path

In this section, we describe the numerical solution of the tariff path with commitment used in Figures 2–4 and Figure 5. Policy choices are compared by discounted home welfare, including the transition.

Step 1. Initial guess. Initially, we guess a path $\{\tau_{21t}\}_{t=0}^{120}$ and set $\tau_{21t} = \tau_{21,120}$ thereafter. The foreign tariff is set to one. For each economy, we set the initial firm stocks to their symmetric free-trade steady-state values:

$$n_{1,-1} = n_{2,-1} = \left\{ \frac{1}{\delta} \left[\frac{b\beta}{\sigma[1 - \beta(1 - \delta)]} \right]^\chi \right\}^{1/(1+\chi)}. \tag{C.1}$$

There are no inherited value multipliers, so we set $\lambda_{i,-1}^V = 0$.

Step 2. Equilibrium for a given tariff path. Prices and profits are evaluated using the static equilibrium formulas in Section 2 at inherited stocks and current tariffs. Stocks start from (C.1),

and, for $i = 1, 2$ and $t = 0, \dots, 219$, stocks and values satisfy

$$\begin{aligned} n_{it} - (1 - \delta)n_{it-1} - (bv_{it})^\chi &= 0, \\ v_{it} - \beta[\pi_{it+1} + (1 - \delta)v_{it+1}] &= 0, \end{aligned} \quad (\text{C.2})$$

and at the terminal date

$$v_{i,220} = \frac{\beta\pi_{i,220}}{1 - \beta(1 - \delta)}, \quad i = 1, 2. \quad (\text{C.3})$$

The system (C.2)–(C.3) is solved jointly by damped Newton iteration with an analytic sparse Jacobian. Stocks and values are scaled by their private free-trade levels; damping preserves positivity. Equilibrium residuals are required to be below an error threshold. The unsaturated-entry restrictions $bv_{it} < 1$ are checked along each reported path.

Step 3. Welfare and the terminal conditions. The numerical approximation to the policy problem is

$$\begin{aligned} \max_{\{\tau_{21t}\}_{t=0}^{120} \in [1, \infty)^{121}} \quad & \sum_{t=0}^{219} \beta^t \left[-\ln P_{1t} + n_{1t-1}\pi_{1t} - \frac{\chi b^\chi}{1 + \chi} v_{1t}^{1+\chi} + \rho R_{1t} \right] \\ & + \frac{\beta^{220}}{1 - \beta} \left[-\ln P_{1,220} + n_{1,219}\pi_{1,220} - \frac{\chi b^\chi}{1 + \chi} v_{1,220}^{1+\chi} + \rho R_{1,220} \right], \end{aligned} \quad (\text{C.4})$$

where $R_{1t} = (1 - \tau_{21t}^{-1})s_{21t}$. The objective in (C.4) includes home profits in both markets, home entry costs, and rebated revenue. Date 220 is included through the term multiplied by $\beta^{220}/(1 - \beta)$ and is not counted in the preceding sum.

Private stocks and values adjust for 100 periods after policy becomes constant. From date 220 on, allocations are approximated by their date-220 values.

Step 4. Tariff path updating. After solving for the equilibrium, welfare is differentiated with respect to each tariff, allowing the equilibrium to respond. The Jacobian of (C.2)–(C.3) is used to compute total tariff derivatives, including the responses of firm values, entry, and stocks. Here $\check{U}_1$ denotes the objective in (C.4). For $t \leq 119$, the derivative of $\check{U}_1$ with respect to τ_{21t} , multiplied by β^{-t} , is

$$\beta^{-t} \frac{d\check{U}_1}{d\tau_{21t}} = \frac{\partial}{\partial \tau_{21t}} [-\ln P_{1t} + n_{1t-1}\pi_{1t} + \rho R_{1t}] + \sum_{i=1}^2 \lambda_{it-1}^v \frac{\partial \pi_{it}}{\partial \tau_{21t}}. \quad (\text{C.5})$$

The partial derivatives on the right hold inherited stocks fixed; this derivative equals the negative of the right-hand side of (A.10). The derivative with respect to $\tau_{21,120}$ instead includes its effects at every date from 120 on, including terminal values and welfare.

We use these welfare derivatives to adjust the tariff path, subject to $\tau_{21t} \geq 1$, and recompute the private equilibrium after each adjustment. We repeat these steps until the first-order conditions below are satisfied.⁹ The first-order conditions for (C.4) are

$$\frac{d\check{U}_1}{d\tau_{21t}} \begin{cases} \leq 0, & \tau_{21t} = 1, \\ = 0, & \tau_{21t} > 1, \end{cases} \quad t = 0, \dots, 120. \quad (\text{C.6})$$

⁹The numerical optimization uses L-BFGS-B (limited-memory Broyden–Fletcher–Goldfarb–Shanno with bound constraints), followed by Newton steps on (C.6) that use the second derivatives of $\check{U}_1$, computed analytically and allowing the equilibrium to respond.